Math 330 Section 2 - Fall 2018 - Homework 07

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Last submission: Friday, September 28, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface, ch.1 – ch.5, ch.6.1 – ch.6.2

MF lecture notes:
ch.1 – ch.3, ch.5, ch.6.1, 6.2, 6.4 – 6.7 (skip ch.6.3)

B/K lecture notes (optional):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments:

Reading assignment 1 - due Monday, September 17:

a. Read carefully B/G ch.6.3 and MF ch.6.8 (Integers modulo n).
b. Review the end of B/G ch.2 (the set S), then read carefully the remainder of B/G ch.6 and MF ch.6.9 and 6.10.

Reading assignment 2 - due: Wednesday, September 19:

a. Read carefully MF ch.6.11. It covers the important parts of the beginning of B/G ch.7 through B/G prop.7.8.
b. Read carefully B/G ch.7.1 (Base–Ten Representation of Integers) prop.7.9 – prop.7.12. Read carefully MF ch.19.7.2 (The Addition Algorithm for Two Nonnegative Numbers (Base 10)).

Note that at this point we are finished with B/G ch.1–ch.7 and with MF ch.1–ch.6 (we skipped the entire chapter 4 on logic).

Reading assignment 3 - due Friday, September 21:

a. Read carefully MF ch.7.1 (The Size of a Set) but SKIP the proof of prop.7.3.
b. Read carefully MF ch.7.2 (The Subsets of $\mathbb{N}$ and Their Size).

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

#1 and #2 are about proving MF thm.6.7 (Division Algorithm for Integers – same as B/G thm.6.13): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

Do not use induction for assignments 1 and 2. It would only make your task more difficult!

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$ such tbat $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints for #2: Review the extended Well-Ordering principle MF thm.6.5. Its use will give the easiest way to prove this assignment: Apply thm.6.5 to the set $A \;:=\; A(m,n) \;:=\; \{x \in \mathbb{Z}_{\geq 0} : x = m - kn$ for some $k \in \mathbb{Z}\}$.

Hint for both #1 and #2: MF prop. 3.51 and cor.3.5 at the end of ch.3 will come in handy in connection with $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following. If $m, n \in \mathbb{Z}_{\geq 0}$ then

$$|n - m| \;\leqq\; \max(m,n), \;\; \text{i.e.,} \tag{3.41}$$

$$-\max(x,y) \;\leqq\; x - y \;\leqq\; \max(x,y), \tag{3.42}$$

$$-n \;<\; y - x \;<\; n. \tag{3.43}$$

Written assignment 3:
Use anything up-to and including MF thm.7.1 and anything in B/G ch.13 to p

Use strong induction to prove MF prop.6.32 on p.143: Any integer $\geqq 2$ has a prime factorization.