Math 330 Section 2 - Fall 2018 - Homework 11

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Update October 15, 2018

Republished the hints to written assignment 1!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface, ch.1 – ch.6, ch.7.1 (only prop.7.9 – prop.7.12), ch.8 – ch.10

MF lecture notes:
ch.1 – ch.3, ch.5, ch.6 (skip ch.6.3), ch.7, ch.8 (skip ch.8.3), ch.9, ch.10.1, ch.10.4,
ch.19.7.2 (The Addition Algorithm for Two Nonnegative Numbers (Base 10))
Any "Addenda" subchapters: those will be added to without notice.

B/K lecture notes (optional):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
- Stewart Calculus 8ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, October 15:

a. Read carefully B/G ch.11., but stop after cor.11.23 in ch.11.3.
b. Read carefully B/G ch.12.1 (Infinite Series).

Reading assignment 2 - due: Wednesday, October 17:

a. Read carefully the remainder of B/G ch.12.

Reading assignment 3 - due Friday, October 19:

a. Read carefully MF ch.10.5. Lots of overlap with B/G ch.12.2!

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

Prove MF thm.10.3: Let $R = (R, +, \cdot, R_{>0})$ be either of the two ordered integral domains $\mathbb{R} = (\mathbb{R}, +, \cdot, \mathbb{R}_{>0})$ or $\mathbb{Q} = (\mathbb{Q}, +, \cdot, \mathbb{Q}_{>0})$. Let $x, y \in R$ such that $x < y$. Then there exists $c \in R$ such that $x < c < y$.

Hint:

Choose $c := \frac{x+y}{2}$. First prove that $2x < x+y < 2y$. What proposition in MF ch.3.4 (Order Relations in Integral Domains) lets you justify this? Then use MF prop.3.??.**??**, also in ch.3.4: $\left[\, a < b \;\&\; c < 0 \;\Rightarrow ac > bc \,\right]$ TWICE IN A ROW to prove that $u < v < w$ implies $\frac{1}{2}u < \frac{1}{2}v < \frac{1}{2}w$.

Written assignment 2: Prove (10.17) of MF prop.10.7: Let X be a nonempty set and $\varphi, \psi : X \to \mathbb{R}$. Let $A \subseteq X$. Then $\inf\{\varphi(x) + \psi(x) : x \in A\} \geqq \inf\{\varphi(y) : y \in A\} + \inf\{\psi(z) : z \in A\}$.

Specific instructions for assignment 2 of this Math 330 homework: Do not follow the MF doc footnote in this proposition (applying $\inf\{\varphi(u) : u \in A\} = -\sup\{-\varphi(v) : v \in A\}$ to (10.16) but do the proof "from scratch", using the proof given for (10.16) as a template.