Math 330 Section 2 - Fall 2018 - Homework 15

Published: Thursday, November 8, 2018 *Running total:* 54 *points*
Last submission: Monday, November 26, 2018

Update November 21, 2018

Instructions were added how to properly do the written assignments.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface, ch.1 – ch.6, ch.7.1 (only prop.7.9 – prop.7.12), ch.8 – ch.10,
ch.11 until before cor.11.23, ch.12 – ch.13.

MF lecture notes:
ch.1 – ch.3, ch.5 – ch.12.2.2, ch.13.1, ch.13.2.1 – ch.13.2.2 (skip ch.6.3 and ch.8.3).
ch.19.7.2 (The Addition Algorithm for Two Nonnegative Numbers (Base 10))
Any "Addenda" subchapters: those will be added to without notice.

B/K lecture notes (optional):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
- Stewart Calculus 8ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- Paul Dawkins' linear algebra lecture notes: As indicated at the bottom of the course materials page

New reading assignments:

Reading assignment 1 - due Monday, November 12:

a. Read carefully B/G Appendix A: Continuity and Uniform Continuity. Note that the only new material is in ch.A.3, and most of that are elementary calculation involving $f(x) = 1/x$ and $f(x) = x^2 + 1$.
b. Read carefully MF ch.13.2.3 – 13.2.4.

Reading assignment 2 - due: Wednesday, November 14:

- Study for the second midterm!

Reading assignment 3 - due Friday, November 16:

a. Read carefully MF ch.13.3.1
b. Read carefully MF ch.13.3.2 through example 13.12 (Alternating series).

Written assignment 1: Prove MF prop.12.13 (Properties of the sup norm): $h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$ defines a norm on $\mathscr{B}(X, \mathbb{R})$

This assignmemt is worth three points: **One point each** for pos.definite, absolutely homogeneous, triangle inequality!

Written assignment 2: Prove MF thm.13.1 (Norms define metric spaces): Let $(V, \|\cdot\|)$ be a normed vector space. Then the function

$$d_{\|\cdot\|}(\cdot,\cdot) : V \times V \to \mathbb{R}_{\geqq 0}; \qquad (x,y) \mapsto d_{\|\cdot\|}(x,y) := \|y - x\|$$

defines a metric space $(V, d_{\|\cdot\|})$.

This assignmemt is worth three points: **One point each** for pos.definite, symmetry, triangle inequality!

Each one of the two assignments of this homework set is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove that $\|\cdot\|_\infty$ satisfies the triangle inequality (12.29c) of a norm you will have to write something along the following lines:

c. Triangle inequality.
NTS: $d(x,y) \leqq d(x,z) + d(z,y)$ for all $x, y, z \in V$.
Proof: $d(x,y) \overset{(\text{def } d_{\|\cdot\|})}{=} \|y - x\| \overset{(\dots)}{=} \cdots \overset{(\dots)}{\leqq} \cdots \overset{(\dots)}{=} d(x,z) + d(z,y)$

Of course you can also write some or all of your proof over several lines and use lengthier explanations or write the numeric references. For example, def. $d_{\|\cdot\|}$ would be (13.3). Thus the last line of the above can also be written as

Proof: $d(x,y) = \|y - x\|$ (definition of $d_{\|\cdot\|}$)
$= \dots$ (.....)
$\leqq \dots$ (.....)
$= d(x,z) + d(z,y)$ (.....)

No need to justify properties of the absolute value $|\alpha|$ of a real number α, but you will need to justify why $\sup\{|\alpha f(x)| : x \in X\} = |\alpha| \sup\{|f(x)| : x \in X\}$ and why $\sup\{|f(x) + g(x)| : x \in X\} \leqq \sup\{|f(x)| : x \in X\} + \sup\{|g(x)| : x \in X\}$.

An aside: **DO NOT** write $\|f(x)\|$ or even $\|f(x)\|_\infty$ when you deal with the real number $f(x)$!

Here are some pointers for assignment 2: You will have to show for each one of (13.1a), (13.1b), (13.1c) how it follows from def. 15: Which one of (12.29a), (12.29b), (12.29c) do you use at which spot?