Math 330 Section 2 - Fall 2018 - Homework 17

Published: Thursday, November 22, 2018 *Running total:* 58 *points*
Last submission: Friday, December 7, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface, ch.1 – ch.6, ch.7.1 (only prop.7.9 – prop.7.12), ch.8 – ch.10,
ch.11 until before cor.11.23, ch.12 – ch.13., Appendix A

MF lecture notes:
ch.1 – ch.3, ch.5 – ch.12.2.2, ch.13
(skip ch.6.3 and ch.8.3).
ch.19.7.2 (The Addition Algorithm for Two Nonnegative Numbers (Base 10))
Any "Addenda" subchapters: those will be added to without notice.

B/K lecture notes (optional):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
- Stewart Calculus 8ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- Paul Dawkins' linear algebra lecture notes: As indicated at the bottom of the course materials page

New reading assignments:

Reading assignment 1 - due Monday, November 26:

- Read carefully MF ch.14.1 – 14.4. SKIP the proof of prop.14.2.

Reading assignment 2 - due: Wednesday, November 28:

- Read carefully MF ch.14.5.

Reading assignment 3 - due Friday, November 30:

- Read carefully MF ch.14.6.

Be sure to follow the supplementary instructions given in hwk 16!

Written assignment 1:
Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$.

Let $f_n : \mathbb{R} \to \mathbb{R}$ be the following sequence of functions:

$$f_n(x) := \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } \frac{-1}{n} \leqq x \leqq 0, \\ -nx + 1 & \text{if } 0 \leqq x \leqq \frac{1}{n}, \end{cases}$$

i.e., the point $\big(x, f_n(x)\big)$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $\frac{-1}{n} \leqq x \leqq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leqq x \leqq \frac{-1}{n}$, and it is on the x–axis for all other x. Draw a picture! Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.

- **a.** Prove that f_n converges pointwise to f on $\mathbb{R}$.
- **b.** Prove that f_n does not converge uniformly to f on $\mathbb{R}$. $\square$

You may use without proof that each of the functions f_n is continuous on $\mathbb{R}$.

Written assignment 2: Given is a metric space (X, d) and two functions $f, g : X \to \mathbb{R}$ which are continuous at $x_0 \in X$. Assume that $g(x_0) \neq 0$. Prove that the quotient $x \to \frac{f(x)}{g(x)}$ is continuous at x_0.

Hint: Study the proofs of prop.13.34 (Rules of arithmetic for continuous real–valued functions).