Math 330 Section 7 - Spring 2019 - Homework 02

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Last submission: Friday, February 8, 2019

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1

MF lecture notes:
ch.1; ch.2.1 – 2.3; ch.3.1
optional - needed for hwk 1: part of ch.3.2

New reading assignments:

OPTIONAL reading assignment (highly recommended)

a. The following parts of the B/K lecture notes are an easy read which covers some of the properties of sets and functions. Read them if you feel wobbly on that subject!

- ch.1.1 (Introduction to sets) (optional)
- ch.1.1 (Introduction to sets) (optional) ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Reading assignment 1 - due Monday, August 27:

a. Read carefully MF ch.3.2 –3.3. Understand that this is the same material as B/G ch.1, except for the notation: $\mathbb{Z}$ in B/G vs. $(R, \oplus, \odot)$ in MF
b. Read carefully B/G ch.2.1 – 2.2.

Reading assignment 2 - due: Wednesday, August 29:

a. Read carefully MF ch.3.4. Understand that this chapter contains to a large degree the same material as B/G ch.2.1 – 2.2.
b. Read carefully B/G ch.2.3 until before prop.2.20
c. Read carefully MF ch.2.4 (induction and recursion). Work through the examples! Finish MF ch.2.

Reading assignment 3 - due Friday, August 31:

a. Read carefully the remainder of MF ch.3.
a. Read carefully the remainder of B/G ch.2. Understand that only with the induction axiom you have the true definition of the integers, and that integral domains need not satisfy it (and the well–ordering principle). Counterexample: $\mathbb{R}$.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

In the written assignments below R denotes an ordered integral domain $(R, \oplus, \odot, P)$.

Written assignment 1:
Use anything up-to and including MF prop. 3.31 to prove MF prop.3.32: The multiplicative unit 1 of R belongs to P.

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

Written assignment 2:
Use anything up-to and including MF prop. 3.32 to prove MF prop.3.33: If $a \in R$ then $a \oplus 1 > a$.

GOOD NEWS: When you do assignments from MF ch.3.4 and later chapters or B/G ch.2 and later chapters, you do not need to justify the rules of arithmetic given to you in MF ch.3.3 and B/G ch.1. No more worry about commutativity of "$\oplus$" and "$\odot$" and the need for parentheses to group more than two terms. You may even use the "general laws of associativity": Given any finite sum of element of an integral domain such as $(a_1 \oplus a_2) \oplus (b_1 \oplus b_2)$ you may regroup the parentheses and even drop them. The same is true for products.