Math 330 Section 7 - Spring 2019 - Homework 06

Published: Friday, February 8, 2019 *Running total: 29 points*
Last submission: Wednesday, February 20, 2019 ***NO RESUBMISSIONS***
(before hwk 5!)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far (includes those of homework 5):

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch5

MF lecture notes:
ch.1 – ch.3; ch.5, ch.6 through ch.6.6 (skim ch.6.3)

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments: None: They came with homework 5.

The written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

Written assignment 1:

Injectivity and Surjectivity

- Let $f : \mathbb{R} \longrightarrow [0, \infty[; \quad x \mapsto x^2$.
- Let $g : [0, \infty[\longrightarrow [0, \infty[; \quad x \mapsto x^2$.
  In other words, g is same function as f as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with **true** or **false**.

a. f is surjective **c.** g is surjective
b. f is injective **d.** g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2:

Find $f : X \longrightarrow Y$ and $A \subseteq X$ such that $f(A^{\complement}) \neq f(A)^{\complement}$. Hint: use $f(x) = x^2$ and choose Y as a **one element only** set (which does not leave you a whole lot of choices for X). See MF example 5.18 on approx. p.113. Start this problem as follows: Let $X := \{......\},\ A := \{......\},\ Y := \{......\}$.

Written assignment 3:

Let $f :]-10, 10[\longrightarrow \mathbb{R}; \quad x \mapsto x^2$.

a. what is the range of f? **b.** Is f injective? **c.** Is f surjective?

d. $f(\{1\} \cup [4,6]) \ =?$ **e.** $f([2,5]) \cap f([4,7]) =?$ **f.** $f^{-1}([4,25]) \cap f^{-1}([16,49]) =?$

Written assignment 4:

You have learned in MF ch.5 that
injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective.

The following illustrates that the reverse is not necessarily true.

Find functions $f : \{a\} \longrightarrow \{b_1, b_2\}$ and $g : \{b_1, b_2\} \longrightarrow \{a\}$ such that $h := g \circ f : \{a\}$ is bijective but such that it is **not true** that both f, g are injective and it is also **not true** that both f, g are surjective. Do not use any other sets when doing this problem!

Hint: There are not a whole lot of possibilities. Draw possible candidates for f and g in arrow notation as on p.118. You should easily be able to figure out some examples. Again, think simple and look at MF example 5.18 on approx. p.113.