Math 330 Section 7 - Spring 2019 - Homework 08

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far:

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch5

MF lecture notes:
ch.1 – ch.3; ch.5 – ch.6 (skim ch.6.3); ch.7.1 – ch.7.2

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments:

Before I lecture about MF ch.7 (Cardinality I: Finite and Countable Sets) I will also talk about some material of B/G ch.6 and 7 which is not part of the MF doc.

Reading assignment 1 - due Monday, February 25:

a. Read carefully B/G ch.6.1 – 6.3. Cross reference the material with MF ch.6.

Reading assignment 2 - due: Wednesday, February 27:

a. Read carefully B/G ch.6.4 and ch.7.1
b. Read carefully MF ch.19.7 (AoP Ch.7: Arithmetic in Base Ten) which contains some addenda to B/G ch.7.

Reading assignment 3 - due Friday, February 29:

a. Read carefully the remainder of MF ch.7 (i.e., ch.7.3)

Written assignments:

You should not attempt to work on assignments 1 and 2 without knowledge of MF ch.6.7 (The Division Algorithm) and you should not attempt to work on assignment 3 without knowledge of MF ch.6.10 (Prime Numbers).

#1 and #2 are about proving MF thm.6.7 (Division Algorithm for Integers – same as B/G thm.6.13): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

Do not use induction for assignments 1 and 2. It would only make your task more difficult!

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$ such tbat $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints for #2: Review the extended Well-Ordering principle MF thm.6.5. Its use will give the easiest way to prove this assignment: Apply thm.6.5 to the set $A \;:=\; A(m,n) \;:=\; \{x \in \mathbb{Z}_{\geq 0} : x = m - kn \text{ for some } k \in \mathbb{Z}\}$.

Hint for both #1 and #2: MF prop. 3.57 and cor.3.5 at the end of ch.3 will come in handy in connection with $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following. If $m, n \in \mathbb{Z}_{\geq 0}$ then

$$|n - m| \;\leqq\; \max(m,n), \quad \text{i.e.,} \tag{3.41}$$
$$-\max(x,y) \;\leqq\; x - y \;\leqq\; \max(x,y), \tag{3.42}$$
$$-n \;<\; y - x \;<\; n. \tag{3.43}$$

Written assignment 3:
Use strong induction to prove MF prop.6.37: Any integer $\geqq 2$ has a prime factorization.