Math 330 Section 7 - Spring 2019 - Homework 14

Published: Thursday, April 4, 2019 *Running total:* 45 *points*
Last submission: Friday, April 19, 2019

Update April 8, 2019

a. *Reading dates were corrected: they erroenously referred to the previous week.*
b. *Adjustment in reading assignments: I repeated for this week the reading assignments from last week, but I added for Friday ch.12.1 ($\mathbb{R}^n$: Euclidean Space).*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far:

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch.6, ch.7.1, ch.8 – ch.13

MF lecture notes:
ch.1 – ch.3; ch.5 – ch.7 (skim ch.6.3); ch.8.1 – 8.2; ch.9.1 through prop.9.7; ch.9.2;
ch.10 – ch.11; ch.19.7(!)

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 8:

a. Read carefully MF ch.10.7. Review the material on indicator functions beforehand!
a. Read carefully MF ch.10.8. To fully appreciate what it says, accept without proof (for now) that $\mathbb{R}$ is uncountable!

Reading assignment 2 - due: Wednesday, April 10:

- Read carefully MF ch.11. Skip the proof of thm.11.4 (the Cantor–Schröder–Bernstein Theorem).

Reading assignment 3 - due Friday, April 12:

- **a.** Read B/G ch.13 on cardinality as follows:
- Skim through ch.13.1 – 13.5. You have seen everything important already in MF ch.7 and MF ch.11.
- Read carefully ch.13.6 on nondescribable numbers, especially if you plan to major in computer science.
- **b.** Skim through MF ch.12.1 ($\mathbb{R}^n$: Euclidean Space). You should be able to grasp the material even if you took neither multivariable calc, nor linear algebra.

Written assignment 1: MF exercise 10.10: Let $x_n := (-1)^n$ for $n \in \mathbb{N}$. Prove that $\liminf_n x_n = -1$ and $\limsup_n x_n = 1$ by working with the tailsets of that sequence. You are not allowed to use anything after def.10.18. **Hint:** What is α_n and β_n?

Written assignment 2 (TWO points): Let $a, b \in \mathbb{R}$ such that $a < b$ and let $(x_n)_n$ be a sequence such that $x_j \in \{a, b\}$ for all j. You may use everything in ch.10.1 – 10.6 to prove the following:

- **a.** If $x_j = a$ eventually then $\limsup\limits_{j\to\infty} x_j = a$.
- **b.** If NOT $x_j = a$ eventually then $\limsup\limits_{j\to\infty} x_j = b$

Hint: Let $A := \{j \in \mathbb{N} : x_j = 1\}$. What can you say about the size of A in case **a**? in case **b**? How does this affect your ability to build subsequences $(x_{n_j})_j$ of $(x_n)_n$ which converge to b?