Math 330 Section 7 - Spring 2019 - Homework 16

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Error in homework 15 reading assignments: I had meant ch.13.1.1 through 13.1.2, NOT 13.1 through 13.2 in Friday's reading assignment.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far:

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch.6, ch.7.1, ch.8 – ch.13

MF lecture notes:
ch.1 – ch.3; ch.5 – ch.7 (skim ch.6.3); ch.8.1 – 8.2; ch.9.1 through prop.9.7; ch.9.2;
ch.10 – ch.11; ch.12.1 – ch.12.2.2; ch.13.1.1 – ch.13.1.2 ch.19.7(!)

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 15:

a. Read ch.13.1.3 – 13.1.5. In particular for convergence: Relate those concepts to the metric spaces $\mathbb{R}$ with metric $d(x,y) = |x-y|$, $\mathbb{R}^n$ with metric $d(\vec{x},\vec{y}) = \|\vec{x}-\vec{x}\|_2$, and $\mathscr{B}(X,\mathbb{R})$ with metric $d(f,g) = \|g-f\|_\infty$.

Reading assignment 2 - due: Wednesday, April 17:

- None: Study for the midterm!

Reading assignment 3 - due Friday, April 19:

- Read carefully MF ch.13.1.6 – ch.13.1.9. Be sure to understand from the definitions(!) that a closed interval in $\mathbb{R}$ contains all its contact points.

Written assignment 1: (3 points!) Prove MF prop.12.13 (Properties of the sup norm): $h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$ defines a norm on $\mathscr{B}(X, \mathbb{R})$

This assignmemt is worth three points: **One point each** for pos.definite, absolutely homogeneous, triangle inequality!

Hint: Go for a treasure hunt in ch.10.2 (Minima, Maxima, Infima and Suprema) and look at the properties of $\sup(A)(A \subseteq \mathbb{R})$ to prove absolute homogeneity and the triangle inequality.

Written assignment 2: (3 points!) Prove MF thm.13.1 (Norms define metric spaces): Let $(V, \|\cdot\|)$ be a normed vector space. Then the function

$$d_{\|\cdot\|}(\cdot,\cdot) : V \times V \to \mathbb{R}_{\geqq 0}; \qquad (x, y) \mapsto d_{\|\cdot\|}(x, y) := \|y - x\|$$

defines a metric space $(V, d_{\|\cdot\|})$.

This assignmemt is worth three points: **One point each** for pos.definite, symmetry, triangle inequality!

Hint for assignment 2: You will have to show for each one of (13.1a), (13.1b), (13.1c) how it follows from def. 12.15: Which one of (12.29a), (12.29b), (12.29c) do you use at which spot?

Each one of the two assignments above is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove that $\|\cdot\|_\infty$ satisfies the triangle inequality (12.29c) of a norm you will have to write something along the following lines:

c. Triangle inequality.
NTS: $d(x, y) \leqq d(x, z) + d(z, y)$ for all $x, y, z \in V$.
Proof: $d(x, y) \overset{(\text{def } d_{\|\cdot\|})}{=} \|y - x\| \overset{(\dots)}{=} \cdots \overset{(\dots)}{\leqq} \cdots \overset{(\dots)}{=} d(x, z) + d(z, y)$

Of course you can also write some or all of your proof over several lines and use lengthier explanations or write the numeric references. For example, def. $d_{\|\cdot\|}$ would be (13.3). Thus the last line of the above can also be written as

Proof: $d(x, y) = \|y - x\|$ (definition of $d_{\|\cdot\|}$)
$= \dots$ (.....)
$\leqq \dots$ (.....)
$= d(x, z) + d(z, y)$ (.....)

No need to justify properties of the absolute value $|\alpha|$ of a real number α, but you will need to justify why $\sup\{|\alpha f(x)| : x \in X\} = |\alpha| \sup\{|f(x)| : x \in X\}$ and why $\sup\{|f(x) + g(x)| : x \in X\} \leqq \sup\{|f(x)| : x \in X\} + \sup\{|g(x)| : x \in X\}$.

An aside: **DO NOT** write $\|f(x)\|$ or even $\|f(x)\|_\infty$ when you deal with the real number $f(x)$!