Math 330 Section 7 - Spring 2019 - Homework 17

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Last submission: Friday, May 10, 2019

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far:

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch.6, ch.7.1, ch.8 – ch.13

MF lecture notes:
ch.1 – ch.3; ch.5 – ch.7 (skim ch.6.3); ch.8.1 – 8.2; ch.9.1 through prop.9.7; ch.9.2;
ch.10 – ch.11; ch.12.1 – ch.12.2.2; ch.13.1.1 – ch.13.1.9; ch.19.7(!)

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 29:

a. Finish ch.13.1: Read carefully ch.13.1.10.
b. Read carefully ch.13.2.1 – ch.13.2.3. Review continuity in ch.10.3 and compare that with ch.13.2.1.

Reading assignment 2 - due: Wednesday, May 1:

a. Read the remainder of ch.13.2.
b. **Optional**: Read B/G appendix A on continuity and uniform continuity.
c. Read carefully ch.13.3.1 and ch.13.3.2 until before remark 13.30.

Reading assignment 3 - due Friday, May 3:

a. Read carefully the remainder of MF ch.13, but skip the proof of Riemann's Rearrangement Theorem.
b. Read MF ch.14.1 – 14.2.

Supplementary instructions for reading MF ch.13:

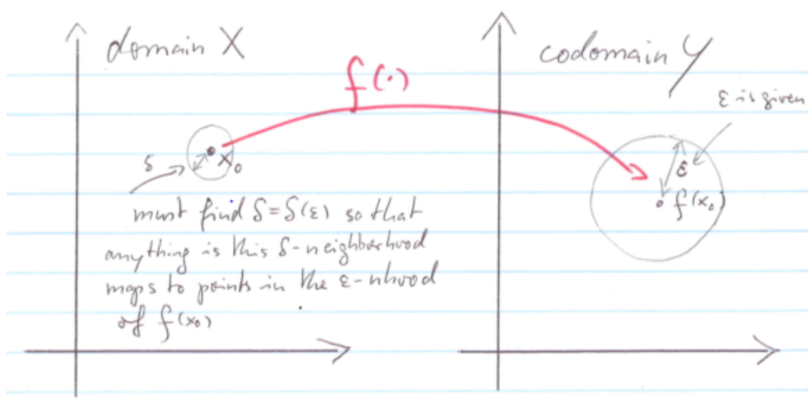
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.13.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for
- open sets and neighborhoods (ch.13.1.3)
- convergence, expressed with nhoods (the end of def.13.10 in ch.13.1.4)

- metric and topological subspaces (ch.13.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \uplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.13.1.9): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
Draw points “completetely inside” B, others “completetely outside” B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 13.6 and thm.13.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.13.2: Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



Written assignments:

You have been taught everything you need to know to solve the following problem in ch.10.3 (Convergence and Continuity in $\mathbb{R}$).

Written assignment 1:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$. You MUST work with def. 13.33 or thm.10.7, NOT with sequence continuity, and you cannot use any "advanced" knowledge such as the product of continuous functions being continuous, etc.

Special instructions for assignment 1: Turn in your scratchpaper where you solve for δ (see the hints below).

Hints:

- **a.** What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
- **b.** $x^2 - 1 = (x+1)(x-1)$.
- **c.** Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then "solving for δ" That part should not be in your official proof.
- **c1.** Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $0 < \varepsilon < 1$. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\, x^2 - 1|$, $|x+1|$, $|x-1|$? if $0 < \delta < 1$?
- **c2.** Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given?
- **c3.** All of the above was done under the assumption that $\delta < 1$ Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$

Written assignment 2: Prove part **d** of MF prop.13.26 (Closure of a set as a hull operator):

Let A be a subset of a topological space $(X, \mathfrak{U})$. Then **a.** $\overline{A \cup B} \subseteq \bar{A} \cup \bar{B}$, **b.** $\bar{A} \cup \bar{B} \subseteq \overline{A \cup B}$.

One point each for **a** and **b**.