Math 330 Section 7 - Spring 2019 - Homework 18

Published: Thursday, May 2, 2019 *Running total:* 58 *points*
Last submission: Friday, May 10, 2019(!!)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far:

B/G (Beck/Geoghegan) Textbook:
Preface and ch.1 – ch.6, ch.7.1, ch.8 – ch.13, appendix A

MF lecture notes:
ch.1 – ch.3; ch.5 – ch.7 (skim ch.6.3); ch.8.1 – 8.2; ch.9.1 through prop.9.7; ch.9.2;
ch.10 – ch.13 (skip ch.12.2.3); ch.14.1 – 14.2 ch.19.7(!)

B/K lecture notes:
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, May 6:

- Read carefully ch.14.3 – ch.14.5, but skip the proof of prop.14.2.

Reading assignment 2 - due: Wednesday, May 8:

a. Read carefully the end of ch.14.
b. Read carefully ch.15.1 – 15.4.

Reading assignment 3 - due Friday, May 10:

- Time off for good behavior

Draw plenty of pictures!

Written assignment 1: One point each for **a** and **b**!

Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$. Let $f_n : \mathbb{R} \to \mathbb{R}$ be the following sequence of functions:

$$f_n(x) \ := \ \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } \frac{-1}{n} \leqq x \leqq 0, \\ -nx + 1 & \text{if } 0 \leqq x \leqq \frac{1}{n}, \end{cases}$$

i.e., the point $\big(x, f_n(x)\big)$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $\frac{-1}{n} \leqq x \leqq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leqq x \leqq \frac{-1}{n}$, and it is on the x–axis for all other x. Draw a picture! Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.

- **a.** Prove that f_n converges pointwise to f on $\mathbb{R}$.
- **b.** Prove that f_n does not converge uniformly to f on $\mathbb{R}$. $\square$

You may use without proof that each of the functions f_n is continuous on $\mathbb{R}$.