Math 330 Section 6 - Fall 2019 - Homework 01

Published: Wednesday, August 21, 2019 *Running total:* 4 *points*
Last submission: Wednesday, September 4, 2019

Update August 28, 2019

*a. Added **c** and **d** to the reading assignment for Friday, August 30*
b. Corrected typo in "Hints for assignments #2 and #3:" ... have to utilize associativity of $\oplus$ to prove #3 and #4... really was meant to be ... have to utilize associativity of $\oplus$ to prove #2 and #3...

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date: None so far since this is your first homework assignment.

New reading assignments:

Reading assignment 1 - due Friday, August 23(!):

- **a.** Read carefully MF ch.2.3 (A First Look at Functions and Sequences) through rem.2.10. Pay particular attention to the motivational note 2.1!
- **b.** Read carefully MF ch.3.1 (Semigroups and Groups) through theorem 3.3.

Reading assignment 2 - due Monday, August 26:

- **a.** Read carefully the remainder of MF ch.3.1.
- **b.** Read carefully MF ch.3.2 (Commutative Rings and Integral Domains).
- **c.** **Optional, but strongly recommended:** Do NOW the following part of Wednesday's reading assignment if you want to start early on the written assignments: Read MF ch.3.3 (Arithmetic in Integral Domains) through MF Prop.3.16 and/or B/G ch.1 through prop.1.11.

Reading assignment 3 - due Wednesday, August 28:

- **a.** Read carefully MF ch.3.3 (Arithmetic in Integral Domains).
- **b.** Read carefully B/G ch.1 (Integers). Study the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 really describe what you know as "integers" or are they more specific or more general than that set of numbers?

Reading assignment 4 - due Friday, August 30:

- **a.** Read carefully MF ch.3.4 (Order Relations in Integral Domains).
- **b.** Read carefully B/G ch.2.1 (Natural Numbers) and ch.2.2 (Ordering the Integers). Study again the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 PLUS ax.2.1 really describe what you know as "integers" or are they more specific or more general than that set of numbers?
- **c.** Read MF ch.2.1 and ch.2.2. You should be familiar with most if not all of this material. You need to be comfortable with the differences between natural numbers, integers, and rational numbers. Note that this material is considered general knowledge for anyone who has studied at least one semester of calculus, a prerequisite for this course. I will skip most of ch.2.1 and 2.2 in class.

d. Optional, but highly recommended if you lack familiarity with basic set operations and functions with arbitrary domain/codomain: Read the following from the B/K (Bryant/Kirby) lecture notes:
- ch.1.1 (Introduction to sets)
- ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Written assignments:

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

For all written assignments here assume that $R = (R, \oplus, \cdot)$ is an integral domain.

> Be sure to use $\oplus$ rather than $+$, $\odot$ rather than $\cdot$, and $\ominus$ rather than $-$. However you may write xy for $x \odot y$ (see MF notations 3.1.**a**). Problem 4 of this homework is an example for this.

Hint for ALL assignments:
Before you start this homework set you should review the entire chapter.3.2 (Commutative Rings and Integral Domains) and then ch.3.3 (Arithmetic in Integral Domains) up to and including prop.3.16 (corresponds to B/G prop.1.11). Look in particular at remark 3.4 which lists in one shot what goes into the definition of an integral domain!

Written assignment 1:
Prove MF Prop.3.15: Let $a, b_1, b_2 \in R$. If $(a \oplus b_1) = 0$ and $a \oplus b_2 = 0$ then $b_1 = b_2$.

Hint: You may use MF prop.3.11 – 3.14.

Hints for assignments #2 and #3:
a. Do **NOT** use commutativity: the variables appear in the same left–to–right order on both sides!
b. Obviously you'll have to utilize associativity of $\oplus$ to prove #2 and #3. Tell me me what you plug in for a, b, c in rem.3.4.**b**.

Written assignment 2:
Prove the first equation of MF Prop.3.16.**b**: Let $a, b, c, d \in R$. Then $a \oplus \big(b \oplus (c \oplus d)\big) \;=\; (a \oplus b) \oplus (c \oplus d)$.

Written assignment 3:
Let $a, b, c, d \in R$. Prove that $\big(a \oplus (b \oplus c)\big) \oplus d \;=\; (a \oplus b) \oplus (c \oplus d)$.

Written assignment 4:
Prove MF Prop.3.16.**d**: Let $a, b, c \in R$. Then $a(bc) \;=\; c(ab)$, i.e., $a \odot (b \odot c) \;=\; c \odot (a \odot b)$.