Math 330 Section 6 - Fall 2019 - Homework 08

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Last submission: Friday, October 11, 2019

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.7 (ch.7 only until thm.7.17)

MF lecture notes:
ch.2, ch.3, ch.5 – ch.6

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 30: (no class)

a. Read carefully MF ch.7.1.

Reading assignment 2 - due: Wednesday, October 2: (midterm 1)

- None: Prepare for the first midterm!

Reading assignment 3 - due Friday, October 4:

a. Read carefully MF ch.7.2 and ch.7.3.

Written assignments:

You should not attempt to work on assignments 1 and 2 without knowledge of MF ch.6.7 (The Division Algorithm) and you should not attempt to work on assignment 3 without knowledge of MF ch.6.10 (Prime Numbers).

#1 and #2 are about proving MF thm.6.7 (Division Algorithm for Integers – same as B/G thm.6.13): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m = n \cdot q + r \quad \text{and } 0 \leq r < n.$$

Do not use induction for assignments 1 and 2. It would only make your task more difficult!

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$ such tbat $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints for #2: Review the extended Well-Ordering principle MF thm.6.5. Its use will give the easiest way to prove this assignment: Apply thm.6.5 to the set $A := A(m,n) := \{r' \in \mathbb{Z}_{\geq 0} : r' = m - q'n$ for some $q' \in \mathbb{Z}\}$. Can you prove that A has a min? Once you have done that what can you do with $\min(A)$?

Hint for both #1 and #2: MF prop. 3.59 and cor.3.5 at the end of ch.3 will come in handy in connection with $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following. If $a, b \in [0, n[_{\mathbb{Z}}$ then

$$|a - b| \leqq \max(a,b), \quad \text{i.e.,} \tag{3.45}$$
$$-\max(a,b) \leqq a - b \leqq \max(a,b), \tag{3.46}$$
$$-n < a - b < n. \tag{3.47}$$