Math 330 Section 6 - Fall 2019 - Homework 14

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Updates November 19, 20, 2019

Hints were added for written assignment 1 on November 19, 2019, and to #2 on November 20, 2019.

Status - Reading Assignments: You were asked to complete the following reading so far:

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.8 (ch.7 only until thm.7.17), ch.13

MF lecture notes:
ch.2, ch.3, ch.5 – ch.10, ch.11, ch.12 through ch.12.1.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit".

New reading assignments:

Reading assignment 1 - due Monday, November 11:

a. Carefully read B/G ch.10. You have encountered that material, some of it in a more abstract setting, in MF ch.3 and ch.9.

Reading assignment 2 - due: Wednesday, November 13:

a. Carefully read B/G ch.9. You have encountered that material, some of it in a more abstract setting, in MF ch.5 and ch.6.16.
b. Carefully read B/G ch.11.1 and 11.2 You have encountered that material in MF ch.9.
c. Skim B/G ch.11.3. This material will not be on any quiz or test.

Reading assignment 3 - due Friday, November 15:

a. Carefully read B/G ch.12. You have encountered that material in MF ch.9.

Written assignment 1: Prove the following which was stated in example 9.9:

If $A_n \subseteq \Omega$ such that $A_n \nearrow$ then $\liminf\limits_{n\to\infty} A_n = \limsup\limits_{n\to\infty} A_n = \bigcup\limits_{n\in\mathbb{N}} A_n$.

Hints:

a) Prove that $\bigcap\limits_{j\geqq n} A_j = A_n$ (easy).

b) Prove that $\bigcup\limits_{j\geqq n} A_j = \bigcup\limits_{j\geqq 1} A_j$ for all n (needs some work). Thus $\bigcup\limits_{j\geqq n} A_j = \dots$

c) Use both **(a)** and **(b)** to prove the limsup result.

d) You only need one of **(a)**, **(b)** to prove the liminf result.

Written assignment 2: Prove prop.10.1:

If X, Y are two sets such that card(X) = card(Y) then card(2^X) = card(2^Y).

Hints:

a) If both $X, Y \neq \emptyset$: Use prop.8.9 + thm.5.1, but NOT on a bijection $X \to Y$ (domain and codomain are 2^X and 2^Y!).

b) If both $X, Y = \emptyset$: Then $2^X =$ _____ $= 2^Y$

c) If one is empty and the other is not: Not possible if card(X) = card(Y) (why)?