Math 330 Section 6 - Fall 2019 - Homework 16

Published: Thursday, November 21, 2019 *Running total:* 56 *points*
Last submission: Monday, December 9, 2019(!!)

Update December 3, 2019

Slight amendment to the formulation of written assignment 2 + last submission date was moved to Mon 12/9.

Status - Reading Assignments: You were asked to complete the following reading so far:

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.13 (ch.7 only until thm.7.17)

MF lecture notes:
ch.2, ch.3, ch.5 – ch.10, ch.11, ch.12 through ch.12.2.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit".

New reading assignments:

Reading assignment 1 - due Monday, November 25:

a. Carefully read the remainder of MF ch.12, including what's not optional in the addenda. Quite a bit in the chapter on infinite series is optional, so you can skip that. Don't try to understand the proof of thm 12.20 (Riemann's Rearrangement Theorem) unless you have a masochistic streak.

Reading assignment 2 - due: Wednesday, November 27: (no class: Thanksgiving break)

a. Carefully read MF ch.13. through ch.13.4 (sequence compactness)

Reading assignment 3 - due Friday, November 29:

a. Carefully read the remainder of MF ch.13.
b. Carefully read MF ch.14 through ch.14.3.

Written assignments:

You have been taught everything you need to know to solve the following problem in ch.9.3 (Convergence and Continuity in $\mathbb{R}$).

Written assignment 1:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$. You MUST work with def. 12.30 or thm.9.8, NOT with sequence continuity, and you cannot use any "advanced" knowledge such as the product of continuous functions being continuous, etc.

Special instructions for assignment 1: Turn in your scratchpaper where you solve for δ (see the hints below).

Hints:

- **a.** What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
- **b.** $x^2 - 1 = (x+1)(x-1)$.
- **c.** Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then "solving for δ" That part should not be in your official proof.
- **c1.** Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $0 < \varepsilon < 1$. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\,x^2 - 1|$, $|x+1|$, $|x-1|$? if $0 < \delta < 1$?
- **c2.** Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given? You'll get the answer by "solving for δ".
- **c3.** All of the above was done under the assumption that $\delta < 1$. Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$
- **d.** Only now you are ready to construct an acceptable proof: Let $\varepsilon > 0, \delta := \dots$, and $\delta' := \min(\delta, 1)$. Then ....

Written assignment 2: Prove part **d** of MF prop.13.26 (Closure of a set as a hull operator):

Let A and B be subsets of a topological space $(X, \mathfrak{U})$. Then **a.** $\overline{A \cup B} \subseteq \bar{A} \cup \bar{B}$, **b.** $\bar{A} \cup \bar{B} \subseteq \overline{A \cup B}$.

One point each for **a** and **b**.