Math 330 Section 6 - Fall 2019 - Homework 17

Published: Wednesday, November 27, 2019 *Running total: 58 points*
Last submission: Monday, December 9, 2019

Update December 3

The written assignment was added on November 27, 2019, and the deadline was shortened to Monday, December 9.

Status - Reading Assignments: You were asked to complete the following reading so far:

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.13 (ch.7 only until thm.7.17)

MF lecture notes:
ch.2, ch.3, ch.5 – ch.13, ch.14 through ch.14.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit".

New reading assignments:

Reading assignment 1 - due Monday, December 2:

a. Read the remainder of MF ch.14 (the optional chapters 14.5 and 14.6). Skip the very technical lemmata 14.3 and 14.4 in ch.14.5.2, but try to get a feeling for the definition of sublinearity in ch.14.5.1. Ch.14.6 on convexity is very brief, and it will give you some new perspective on the definition of concave–up functions.

No further reading assignment! Prepare for your finals!

Written assignment 1: One point each for **a** and **b**!

Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$. Let $f_n : \mathbb{R} \to \mathbb{R}$ be the following sequence of functions:

$$f_n(x) \;:=\; \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } \frac{-1}{n} \leqq x \leqq 0, \\ -nx + 1 & \text{if } 0 \leqq x \leqq \frac{1}{n}, \end{cases}$$

i.e., the point $\big(x, f_n(x)\big)$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $\frac{-1}{n} \leqq x \leqq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leqq x \leqq \frac{-1}{n}$, and it is on the x–axis for all other x. Draw a picture! Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.

a. Prove that f_n converges pointwise to f on $\mathbb{R}$.
b. Prove that f_n does not converge uniformly to f on $\mathbb{R}$. $\square$

You may use without proof that each of the functions f_n is continuous on $\mathbb{R}$.