Math 330 Section 6 - Spring 2020 - Homework 01

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As of January 19, the references given here such as "MF Prop.3.17" refer to version 2020.01.18 and ***NO MORE*** *to version 2019-12-03 of the MF doc!*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date: None so far since this is your first homework assignment.

New reading assignments:

Reading assignment 1 - due Friday, January 24(!):

- **a.** Review MF ch.2.1 (Sets and Basic Set Operations) and ch.2.2 (Numbers). Those chapters are meant for self–study since most if not all of the material found there should be familiar to you. You need to be comfortable with the differences between natural numbers, integers, and rational numbers. Note that this material is considered general knowledge for anyone who has studied at least one semester of calculus, a prerequisite for this course. I will skip most of ch.2.1 and 2.2 in class.
- **b.** Read carefully MF ch.2.3 (A First Look at Functions and Sequences). Pay particular attention to the motivational note 2.1!
- **c.** Read carefully MF ch.3.1 (Semigroups and Groups) through theorem 3.3.

Reading assignment 2 - due Monday, January 27:

- **a.** Read carefully the remainder of MF ch.3.1.
- **b.** Read carefully MF ch.3.2 (Commutative Rings and Integral Domains).
- **c.** **Optional, but strongly recommended:** Do NOW the following part of Wednesday's reading assignment if you want to start early on the written assignments: Read MF ch.3.3 (Arithmetic in Integral Domains) through MF Prop.3.16 (barely more than $1\frac{1}{2}$ pages) and/or B/G ch.1 through prop.1.11.

Reading assignment 3 - due Wednesday, January 29:

- **a.** Read carefully MF ch.3.3 (Arithmetic in Integral Domains).
- **b.** Read carefully B/G ch.1 (Integers). Study the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 really describe what you know as "integers" or are they more specific or more general than that set of numbers?

Reading assignment 4 - due Friday, January 31:

- **a.** Read carefully MF ch.3.4 (Order Relations in Integral Domains).
- **b.** Read carefully B/G ch.2.1 (Natural Numbers) and ch.2.2 (Ordering the Integers). Study again the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 PLUS ax.2.1 really describe what you know as "integers" or are they more specific or more general than that set of numbers?
- **c.** <u>Optional, but highly recommended</u> if you lack familiarity with basic set operations and functions with arbitrary domain/codomain: Read the following from the B/K (Bryant/Kirby) lecture notes:
  - ch.1.1 (Introduction to sets)
  - ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

For all written assignments here assume that $R = (R, \oplus, \cdot)$ is an integral domain.

Be sure to use $\oplus$ rather than $+$, $\odot$ rather than $\cdot$, and $\ominus$ rather than $-$. However you may write xy for $x \odot y$ (see MF notations 3.1.**a**). Problem 4 of this homework is an example for this.

Hint for ALL assignments:
Before you start this homework set you should review the entire chapter.3.2 (Commutative Rings and Integral Domains) and then ch.3.3 (Arithmetic in Integral Domains) up to and including prop.3.17 (corresponds to B/G prop.1.11). Look in particular at remark 3.4 which lists in one shot what goes into the definition of an integral domain!

Written assignment 1:
Prove MF Prop.3.16: Let $a, b_1, b_2 \in R$. If $a \oplus b_1 = 0$ and $a \oplus b_2 = 0$ then $b_1 = b_2$.

Hint: You may use MF prop.3.12 – 3.15.

Hints for assignments #2 and #3:
a. Do **NOT** use commutativity: the variables appear in the same left–to–right order on both sides!
b. Obviously you'll have to utilize associativity of $\oplus$ to prove #2 and #3. Tell me me what you plug in for a, b, c in rem.3.4.**b**.

Written assignment 2:
Prove the first equation of MF Prop.3.17.**b**: Let $a, b, c, d \in R$. Then $a \oplus \big(b \oplus (c \oplus d)\big) \;=\; (a \oplus b) \oplus (c \oplus d)$.

Written assignment 3:
Let $a, b, c, d \in R$. Prove that $\big(a \oplus (b \oplus c)\big) \oplus d \;=\; (a \oplus b) \oplus (c \oplus d)$.

Written assignment 4:
Prove MF Prop.3.17.**d**: Let $a, b, c \in R$. Then $a(bc) \;=\; c(ab)$, i.e., $a \odot (b \odot c) \;=\; c \odot (a \odot b)$.

Do not forget written assignment zero:

Send an email by Friday, January 24, that
a. lists your math background, including for the current semester(!),
b. acknowledges that you have read the syllabus posted on the course website and/or on Blackboard,
c. tells me why you chose to take this course