

Math 330 Section 6 - Spring 2020 - Homework 03

Published: Monday, January 28, 2020

Running total: 17 points

Last submission: Friday, February 7, 2020

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:

ch.1, ch.2.1 – 2.2

MF lecture notes:

ch.2 through ch.2.3, ch.3.1 – 3.4

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, February 3:

- a. Carefully read MF ch.3.5 (Minima, Maxima, Infima and Suprema in Ordered Integral Domains). Be sure you understand what the material discussed there means for the sets \mathbb{Z} , \mathbb{Q} and \mathbb{R} .
- b. Be aware of what is in the addenda to ch.3. The material was already taught in week 2 of the semester.
- c. Read the preface and the notes for both student and instructor in the B/G (Beck Geoghegan) text.
- d. Read carefully B/G ch.3 about logic. It is very brief. In particular familiarize yourself with the different english expressions for “if p then q” and the negation of statements with quantifiers.

Reading assignment 2 - due: Wednesday, February 5:

- a. Read the remainder of MF ch.2.3. You need to have a basic understanding of why an expression like $a + b$ can be thought of as a function of two variables a and b .
- b. Read **extra carefully** MF ch.2.4 about induction and recursion. Proofs that use the induction principle are guaranteed to be on both midterms and also the final exam! Use pencil and paper and work through each one of the proofs given in this short chapter!
- c. Read the remainder of MF ch.2 (i.e., ch.2.5: Some Preliminaries From Calculus). All of it should be familiar to you.
- d. Look at the sample homework assignment which is linked to from the Homework page of the course website. It shows you how to use proper margins, double spacing, etc.

Reading assignment 3 - due Friday, February 7:

- a. Carefully read MF ch.5.1 (Cartesian Products and Relations). It is rather abstract.
- b. Carefully read MF ch.5.2.1 (Some Preliminary Observations about Functions). and MF ch.5.2.2. (Definition of a Function and Some Basic Properties). Read at least the easier examples given for functions in MF ch.5.2.3 (Examples of Functions)

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

In the written assignments below R denotes an ordered integral domain (R, \oplus, \odot, P) .

Written assignment 1:

Use anything up-to and including MF prop. 3.34 to prove MF prop.3.35: The multiplicative unit 1 of R belongs to P .

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

Written assignment 2:

Use anything up-to and including MF prop. 3.39 to prove MF prop.3.40: If $a, b \in R$ and $a \leq b \leq a$ then $a = b$.

Written assignment 3:

Use anything up-to and including MF prop. 3.49 to prove MF prop.3.50: If $a \in R$ then $|a|^2 = a^2$.

GOOD NEWS: When you do assignments from MF ch.3.4 and later chapters or B/G ch.2 and later chapters, you do not need to justify the rules of arithmetic given to you in MF ch.3.3 and B/G ch.1. No more worry about commutativity of " \oplus " and " \odot " and the need for parentheses to group more than two terms. You may even use the "general laws of associativity": Given any finite sum of element of an integral domain such as $(a_1 \oplus a_2) \oplus (b_1 \oplus b_2)$ you may regroup the parentheses and even drop them. The same is true for products.