# Math 330 Section 6 - Spring 2020 - Homework 08

*Published: Thursday, February 20, 2020* *Running total:* 35 *points*
*Last submission: Friday, March 13, 2020*

**Status - Reading Assignments:**

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 – 6

MF lecture notes:
ch.2, ch.3, ch.5 – 8 (not the optional ch.8.3 and 8.5)

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

**New reading assignments:**

**Reading assignment 1 - due Monday, March 2:**

**a.** Prepare for today's midterm.
**b.** Carefully read B/G ch.7.1, ch.7.2 until thm.7.17 (skip the proof), and ch.9.

**Reading assignment 2 - due: Wednesday, March 4:**

**a.** Carefully read MF ch.9.1 – 9.2.

**Reading assignment 3 - due Friday, March 6:** (no class)

**a.** Carefully read B/G ch.8.
**b.** Carefully read B/G ch.10 through prop.10.13 in ch.10.4.
**c.** Carefully read MF ch.9.3 **(VERY IMPORTANT!)**

Written assignments are on the next page.

**Written assignments:**

You should not attempt to work on assignments 1 and 2 without knowledge of MF ch.6.9 (The Division Algorithm).

#1 and #2 are about proving MF thm.6.9 (Division Algorithm for Integers – same as B/G thm.6.13): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers $q$ ("quotient") and $r$ ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

| Do not use induction for assignments 1 and 2. It would only make your task more difficult! |
|---|

**Written assignment 1:**

Prove uniqueness of the "decomposition" $m = qn + r$ such tbat $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

**Written assignment 2:**

Much harder than #1: Prove the existence of $q$ and $r$.

**Hints for #2**: Review the extended Well-Ordering principle MF thm.6.7. Its use will give the easiest way to prove this assignment: Apply thm.6.7 to the set $A \;:=\; A(m,n) \;:=\; \{r' \in \mathbb{Z}_{\geq 0} : r' = m - q'n$ for some $q' \in \mathbb{Z}\}$. Can you prove that $A$ has a min? Once you have done that what can you do with $\min(A)$?

**Hint for both #1 and #2**: MF prop. 3.61 and cor.3.5 at the end of ch.3 will come in handy in connection with $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following. If $a, b \in [0, n[_{\mathbb{Z}}$ then

$$|a - b| \;\leqq\; \max(a,b), \;\; \text{i.e.,} \tag{3.46}$$
$$-\max(a,b) \;\leqq\; a - b \;\leqq\; \max(a,b), \tag{3.47}$$
$$-n \;<\; a - b \;<\; n. \tag{3.48}$$