Math 330 Section 6 - Spring 2020 - Homework 11

Published: Sunday, March 22, 2020 *Running total:* 41 *points*
Last submission: Friday, April 3, 2020

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 – 13 (ch.7.2 until thm.7.17),

MF lecture notes:
ch.2, ch.3, ch.5 – 11

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, March 23:

- No reading assignment

Reading assignment 2 - due: Wednesday, March 25:

a. Carefully read MF ch.12.1.1 – 12.1.2. You need to remember what you studied about norms in ch.11.2.2!

Reading assignment 3 - due Friday, March 27:

a. Carefully read MF ch.12.1.3 – 12.1.4. Draw plenty of pictures!

Supplementary instructions for reading MF ch.12:

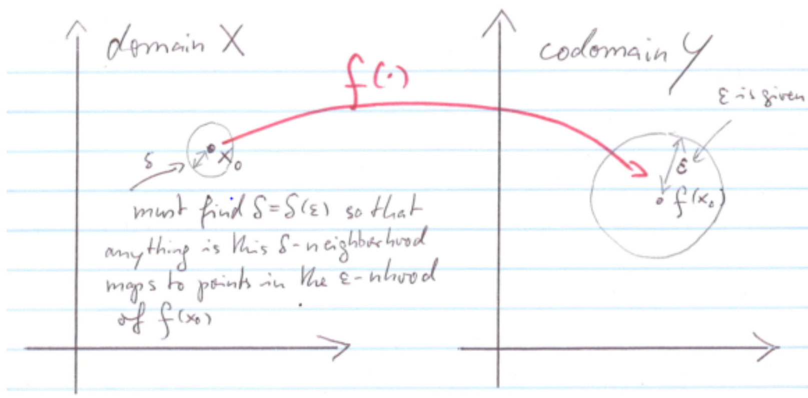
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for
- open sets and neighborhoods (ch.12.1.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
- metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What is $N_\varepsilon^A(x_j)$?

- Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.12.2: Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



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Written assignments:

New policy:

Your written assignments must be submitted as .txt files or .xdoc files. If you know how to, you may also submit them as LaTex files. In that case I want you to submit the .tex source file. I should be able to run your xyz.tex file as "pdflatex xyz". You can find a link to the instructions on the Homework page.

Written assignment 1: Prove MF prop.9.17.**b**: Let y_n be a sequence of real numbers that is nonincreasing, i.e., y_n >= y_(n+1) for all n, and bounded below.

Then lim[y_n : n -> infty] exists and coincides with inf[y_n : n in N].

Do the proof by modifying the proof of prop.9.17.**a**. You are **NOT ALLOWED** to apply prop.9.17.**a** to the sequence x_n := -y_n!

Written assignment 2: Prove MF thm. 9.8: If m in N is not a perfect square then sqrt(m) is irrational.