Math 330 Section 6 - Spring 2020 - Homework 13

Published: Thursday, April 9, 2020 *Running total:* 45 *points*
Last submission: Friday, April 24, 2020

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 – 13 (ch.7.2 until thm.7.17),

MF lecture notes:
ch.2, ch.3, ch.5 – 12.2.1

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, April 13:

a. Carefully read the remainder of MF ch.12.2.
b. Carefully read MF ch.12.3 through def.12.40 (absolutely convergent series).

Reading assignment 2 - due: Wednesday, April 15:

a. Prepare for the second midterm. Scope is MF ch.7 – ch.10. The midterm will consist of five proofs that you must write.

Reading assignment 3 - due Friday, April 17:

a. Carefully read the remainder of MF ch.12. That is very little, since most of it consists of the optional lemma 12.1 and the optional proof of Riemann's Rearrangement Theorem.
b. Carefully read MF ch.13 through the end of ch.13.3 Note that most of ch.13.3 consists of the optional prop.13.2.

Written assignments:

New policy:

Your written assignments must be submitted as .txt files or .xdoc files. If you know how to, you may also submit them as LaTex files. In that case I want you to submit the .tex source file. I should be able to run your xyz.tex file as "pdflatex xyz". You can find a link to the instructions on the Homework page.

Written assignment 1: Prove prop.10.1:

If X, Y are two sets such that card(X) = card(Y) then card(2 ^ X) = card(2 ^ Y).

Hints:

a) If both X, Y are not empty: Use prop.8.9 + thm.5.1, but use the latter NOT on a funtion with domain X and codomain Y (here domain and codomain are 2 ^ X and 2 ^ Y!).

b) If both X, Y are empty: Then 2 ^ X = _____ = 2 ^ Y

c) If one of X, Y is empty and the other is not: Not possible if card(X) = card(Y). (WHY)?

Written assignment 2 (3 points!): Prove MF prop.11.13 (Properties of the sup norm):

h –> norm(h) := sup { | h(x) | : x in X }

defines a norm on the set B(X, R) of all real-valued, bounded functions.

This assignment is worth three points: **One point each** for pos.definite, absolutely homogeneous, triangle inequality!

Hint: Go for a treasure hunt in ch.9.2 (Minima, Maxima, Infima and Suprema) and look at the properties of $\sup(A)(A \subseteq \mathbb{R})$ to prove absolute homogeneity and the triangle inequality.

Assignment 2 above is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove that norm(h) satisfies the triangle inequality (11.29c) of a norm you will have to write something along the following lines:

c. Triangle inequality.
NTS: norm(f + g) <= norm(f) + norm(g) for all f, g in B(X, R).
Proof: norm(f + g) = sup { ..... } (def. norm(.))
<= sup { ..... } + sup { ..... } (prop.9....)
= norm(f) + norm(g) (def. norm(.)) .

No need to justify properties of the absolute value $|\alpha|$ of a real number α, but you will need to justify why $\sup\{|\alpha f(x)| : x \in X\} = |\alpha| \sup\{|f(x)| : x \in X\}$ and why $\sup\{|f(x) + g(x)| : x \in X\} \leqq \sup\{|f(x)| : x \in X\} + \sup\{|g(x)| : x \in X\}$.

An aside: **DO NOT** write norm(f(x)) when, in reality, you deal with the real number | f(x) | !