Math 330 Section 6 - Spring 2020 - Homework 14

Published: Friday, April 17, 2020 *Running total:* 49 *points*
Last submission: Friday, May 1, 2020

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 – 13 (ch.7.2 until thm.7.17),

MF lecture notes:
ch.2, ch.3, ch.5 – 13.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, April 20:

- **a.** Carefully read B/G Further Topic A: Continuity and Uniform Continuity. You alreadg know the material, but be sure that you can match it up with the corresponding items in the MF doc.
- **b.** Carefully read MF ch.13.4 and 13.5.
- **c.** Those of you who did not take a linear algebra should revisit the last pages of MF ch.11.2.1 where I discuss linear independence and bases of vector spaces.

Reading assignment 2 - due: Wednesday, April 22:

- **a.** Carefully read the remainder of MF ch.13.
- **b.** Carefully read MF ch.14.1 – 14.4.

Reading assignment 3 - due Friday, April 24:

- **a.** Read the optional(!) MF ch.6.5, but skip the proof of prop.6.16 unless you would like to see some nontrivial usage of binomial coefficients.
- **b.** Carefully read prop.12.44 of MF ch.12.3.1 about the uniform convergence of certain Bernstein polynomials.
- **c.** Carefully read MF ch.15 through the end of ch. 15.1.

Written assignments:

New policy:

Your written assignments must be submitted as .txt files or .xdoc files. If you know how to, you may also submit them as LaTex files. In that case I want you to submit the .tex source file. I should be able to run your xyz.tex file as "pdflatex xyz". You can find a link to the instructions on the Homework page.

Written assignment 1: Prove MF thm.12.1 (Norms define metric spaces): Let V be a vector space with a norm norm(.).

Then the function d : V x V –> [0, infty[(x,y) -> d(x, y) := norm(y - x) defines a metric space (V,d).

This assignment is worth three points: **One point each** for pos.definite, symmetry, triangle inequality!

Hint: You will have to show for each one of (12.1a), (12.1b), (12.1c) how it follows from def. 11.15: Which one of (11.31a), (11.31b), (11.31c) do you use at which spot?

The assignment above is worth three points, and you will have to earn them! The following exemplifies the level of detail I expect you to provide.

To prove that the function d satisfies the triangle inequality of a metric you will have to write something along the following lines:

c. Triangle inequality.
NTS: d(x, y) <= d(x, z) + d(z, y) for all x, y, z in V.
Proof: d(x, y) = norm(y - x) (definition of the metric d)
= . . . (.....)
<= . . . (.....)
= d(x, z) + d(z, y) (.....)

Written assignment 2: Prove prop.12.12 of ch.12.1.5 (Abstract Topological spaces): Let (X,d) be a metric space with the discrete metric d(x, x′) = 0 if x = x′ and 1 if x <> y. (“<>” stands for “not equal to”).

Then its associated topology (the discrete topology) is 2^X = { A : A subset of X }.