Math 330 Section 6 - Spring 2020 - Homework 15

Published: Thursday, April 23, 2020 *Running total:* 52 *points*
Last submission: Tuesday, May 5, 2020 *(last day of classes)*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 – 13 (ch.7.2 until thm.7.17), Further Topic A: Continuity and Uniform Continuity

MF lecture notes:
ch.2, ch.3, ch.5 – 14.4, ch.15.1, the optional stuff on Bernstein polynomials

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Reading assignment 1 - due Monday, April 27:

a. I overlooked the following when doing the previous homework sheet: Read MF ch.14.5, but skip the two tecnnical lemmata 14.3 and 14.4.
b. MF ch.14.6: SKIP IT!

Reading assignment 2 - due: Wednesday, April 29:

a. Carefully read MF ch.15.2. The proof are lengthy, but not overly challenging.

Reading assignment 3 - due Friday, May 1:

a. Carefully read the remainder of MF ch.15 (ch.15.3).

Written assignments:

New policy:

Your written assignments must be submitted as .txt files or .xdoc files. If you know how to, you may also submit them as LaTex files. In that case I want you to submit the .tex source file. I should be able to run your xyz.tex file as "pdflatex xyz". You can find a link to the instructions on the Homework page.

You have been taught everything you need to know to solve the following problem in ch.9.3 (Convergence and Continuity in $\mathbb{R}$).

Written assignment 1:

Let f(x) = x^2. Prove by use of "ε-δ continuity" (suggestion: write eps and delta) that f is continous at x_0 = 1. You MUST work with def. 12.30, NOT with sequence continuity, and you cannot use any "advanced" knowledge such as the product of continuous functions being continuous, etc.

Special instructions for assignment 1: Turn in your scratchpaper (hand–written!) where you solve for δ (see the hints below).

Hints:

a. What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?

b. $x^2 - 1 = (x+1)(x-1)$.

c. Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then "solving for δ" That part should not be in your official proof.

c1. Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $0 < \varepsilon < 1$. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\,x^2 - 1|$, $|x+1|$, $|x-1|$? if $0 < \delta < 1$?

c2. Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given? You'll get the answer by "solving for δ".

c3. All of the above was done under the assumption that $\delta < 1$. Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$

d. Only now you are ready to construct an acceptable proof: Let $\varepsilon > 0, \delta := \dots$, and $\delta' := \min(\delta, 1)$. Then ....

Written assignment 2: Prove part **d** of MF prop.12.27 (Closure of a set as a hull operator):

Let A and B be subsets of a topological space X. Then

a. closure(A U B) subset of closure(A) U closure(B),

b. closure(A) U closure(B) subset of closure(A U B).

Reminder: "U" stands for "union"

One point each for **a** and **b**.