Math 330, Section 6
HW 10 – Version 2 – September 5, 2020
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@@MF: Note to the student:

a. I will tag my comments with **@@MF:** so you can easily recognize them.
b. Observe that this newer version 2 goes on top of the older version 1 (see below!)

Written Assignment 1:

Let A be a nonempty set and let $S := \{f : f \text{ is a function } A \to A\}$.

Let $\circ$ be the following binary operation on S: $(f, g) \mapsto g \circ f$ assigns to two functions $f, g : A \to A$ the function

$$g \circ f : A \to A; \qquad x \mapsto g \circ f(x) := g\big(f(x)\big).$$

Prove that $(S, \circ)$ is a monoid.

PROOF:

We need to show that $\circ$ is associative and that S contains a neutral element.

A. Proof of associativity.

Let $f, g, h \in S$ and $x \in A$. It follows from the definition of $\circ$ that

$$\big((h{\circ}g){\circ}f\big)(x) = (h{\circ}g)\big(f(x)\big) = h(g(f(x)) = h\big((g{\circ}f)(x)\big) = \big(h{\circ}(g{\circ}f)\big)(x).$$

Since $\big((h \circ g) \circ f\big)(x) = \big(h \circ (g \circ f)\big)(x)$ for each argument x it follows that the functions $(h \circ g) \circ f$ and $h \circ (g \circ f)$ coincide. This proves associativity.

B. Existence of a neutral element.

Let $id_A : A \to A;\ x \mapsto x$ be the identity function on A. Then

$$\begin{aligned}
\big(id_A \circ f\big)(x) &= id_A\big(f(x)\big) && \text{(def. of } \circ) \\
&= f(x) && \text{(def. of } id_A) \\
&= f\big(id_A(x)\big) && \text{(def. of } id_A) \\
&= \big(f \circ id_A\big)(x) && \text{(def. of } \circ)
\end{aligned}$$

for all $x \in A$.

It follows that

$$\big(id_A \circ f\big)(x) \ = f(x) \ = \ \big(f \circ id_A\big)(x)$$

for all $x \in A$, thus the functions $id_A \circ f$, f, and $f \circ id_A$ are equal. This proves that id_A is a neutral element for $(S, \circ)$.). ■

Math 330, Section 6

HW 10 – Version 1 – September 5, 2020

Author: Antonio Sample

@@MF: Note to the student:

Observe the "newpage" command which forces a page break to separate the newer version 2 (above!) from the older version 1 (below!) and that the entire header info is repeated.

Written Assignment 1:

Let A be a nonempty set and let $S := \{f : f \text{ is a function } A \to A\}$.

yada yada ....