Math 330, Section 6

HW 13 – Version 1 – September 5, 2020
Author: Bernice Sample

Written Assignment 1:

Let $(R, \oplus, \odot)$ be a commutative ring with unit. Then $a \odot 0 = 0$ for all $a \in R$.

PROOF – version 1:

Note to the reader: This version of the proof shows you how to how to use the stackrel feature to provide references above the equals sign. The references may be off since the version of the MF doc on which they are based is an older one.

$$\begin{aligned} a \odot 0 &\stackrel{(3.2)}{=} a(0 \oplus 0) \\ &\stackrel{(3.16)}{=} a \odot 0 \oplus a \odot 0. \qquad (\star) \end{aligned}$$

Hence

$$\begin{aligned} 0 &\stackrel{(3.5)}{=} a \odot 0 \ \oplus \big(\ominus (a \odot 0)\big) \\ &\stackrel{(\star)}{=} (a \odot 0 \oplus a \odot 0) \ \oplus \big(\ominus (a \odot 0)\big) \\ &\stackrel{(3.1)}{=} a \odot 0 \ \oplus \ \big(a \odot 0) \ \oplus (\ominus(a \odot 0)\big) \\ &\stackrel{(3.5)}{=} a \odot 0 \oplus 0 \\ &\stackrel{(3.2)}{=} a \odot 0. \end{aligned}$$

This proves that $a \odot 0 = 0$. ■

PROOF – version 2:

Note to the reader: This version of the proof shows you an alternate version: provide the references to the right. Again, the references may be off since the version of the MF doc on which they are based is an older one.

$$\begin{aligned} a \odot 0 &= a(0 \oplus 0) && \text{– use (3.2)} \\ &= a \odot 0 \oplus a \odot 0. && \text{– use (3.16)} \qquad (\star) \end{aligned}$$

Hence

$$
\begin{aligned}
0 &= a \odot 0 \ \oplus \big(\ominus (a \odot 0)\big) && \text{– use (3.5)}\\
&= (a \odot 0 \oplus a \odot 0) \ \oplus \big(\ominus (a \odot 0)\big) && \text{– use } (\star)\\
&= a \odot 0 \ \oplus \ \big(a \odot 0) \ \oplus (\ominus(a \odot 0)\big) && \text{– use (3.1)}\\
&= a \odot 0 \oplus 0 && \text{– use (3.5)}\\
&= a \odot 0. && \text{– use (3.2)}
\end{aligned}
$$

This proves that $a \odot 0 = 0$. ■