Math 330 Section 6 - Fall 2020 - Homework 06

Published: Thursday, September 17, 2020 *Running total: 29 points*
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Update September 19, 2020

Complemented Monday's reading assignment with some HTML links for $\sum$ notation tutorials.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1, ch.2.1 - 2.3, ch.3, ch.5

MF lecture notes:
ch.2 - 3, ch.5, ch.6.1 - 6.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 21:

a. Familiarize your self with the $\sum$ notation $\sum\limits_{j=m}^{n} x_j$ for the sum $x_m + x_{m+1} + \cdots + x_{n-1} + x_n$ of a finite or infinite sequence of numbers x_j. Here are some links you may want to look at if you have not worked with the $\sum$ (Sigma) symbol before:
- `http://onlinestatbook.com/2/introduction/summation.html`
- `http://www.columbia.edu/itc/sipa/math/summation.html`
- `http://www.purplemath.com/modules/series.htm`
- `http://www.purplemath.com/modules/series2.htm`
- `http://study.com/academy/lesson/summation-notation-rules-examples-quiz.html#lesson`

b. Carefully read MF ch.6.6-6.9.

Reading assignment 2 - due: Wednesday, September 23:

a. Carefully read B/G ch.2.4 and B/G ch.6.1-6.2 You have encountered the material already in MF ch.5.1 and ch.6.8-6.9.
b. Carefully read MF ch.6.10-6.12

Reading assignment 3 - due Friday, September 25:

a. Carefully read the remainder of MF ch.6.
b. Carefully read the remainder of B/G ch.6 and then ch.7.1.

Written assignments are on the next page.

Written assignment 1:
Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$ according to MF def.6.11 in ch.6.6 (Divisibility) or the definitions that follow B/G prop.1.14.

Written assignment 2:
Prove MF Prop. 6.7**a** by induction on p: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^{p} x_j \;=\; \sum_{j=m}^{n} x_j \;\oplus\; \sum_{j=n+1}^{p} x_j .$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p? If $m = -4$ and $n = 8$, how would you choose p? For general $m \leqq n$, how would you choose p?

Written assignment 3:
Let $x_0 = 8,\ x_1 = 16,\ x_{n+1} = 6x_{n-1} - x_n$ for $n \in \mathbb{N}$.

Prove that $x_n = 2^{n+3}$ for every integer $n \geq 0$.

Hint: Use strong induction.