Math 330 Section 6 - Fall 2020 - Homework 07

Published: Thursday, September 24, 2020 *Running total: 32 points*
Last submission: Friday, October 16, 2020

Update October 14, 2020

Deadline moved from 10/16 to 10/23.

Update September 30, 2020

Added the written assignments. Deadline moved from 10/9 to 10/16.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - 4, ch.3, ch.6 - 7.1

MF lecture notes:
ch.2 - 3, ch.5 - 6

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 28:

- **a.** Carefully read B/G ch.4.1 - 4.5 and skim ch.4.6. You have seen all this material in MF ch.6.
- **b.** Carefully read B/G ch.5. The few things you will find there have been covered in MF ch.5.

Reading assignment 2 - due: Wednesday, September 23:

- **a.** Carefully read MF ch.7.1-7.3.

Reading assignment 3 - due Friday, September 25:

- **a.** Carefully read the remainder of MF ch.7.
- **b.** Carefully read MF ch.8.1, skim ch.8.2, and then carefully read MF ch.8.3, in particular, def.8.6 and the subsequent remarks. (Cartesian Product of a family of sets).

Written assignments are on the next page.

Written assignments:

#2 and #3 are about proving MF thm.6.8 (Division Algorithm for Integers – same as B/G thm.6.13): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \,=\, n \cdot q \,+\, r \quad \text{and } 0 \leq r < n.$$

Do not use induction for assignments 2 and 3. It would only make your task more difficult!

Written assignment 1:
Prove prop.5.9.**a**: Let $X, Y \neq \emptyset$, and $f : X \to Y$ bijective. Let $\emptyset \neq A \subseteq X$, $B := f\big|_A(A) = \{f(a) : a \in A\}$. [1]
Let $f' : A \to B; \quad x \mapsto f(x)$, i.e., $f' = f\big|_A$, except that we have shrunken the codomain Y to B. Then f' is bijective.

Written assignment 2:

Prove uniqueness of the "decomposition" $m = qn + r$ such tbat $0 \leq r < n$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 3:

Much harder than #2: Prove the existence of q and r.

Hints for #3: Review the extended Well-Ordering principle MF thm.6.7. Its use will give the easiest way to prove this assignment: Apply thm.6.7 to the set $A := A(m,n) := \{r' \in \mathbb{Z}_{\geq 0} : r' = m - q'n \text{ for some } q' \in \mathbb{Z}\}$. Can you prove that A has a min? Once you have done that what can you do with $\min(A)$?

Hint for both #2 and #3: MF prop. 3.61 and cor.3.5 at the end of ch.3.5 will come in handy in connection with using or proving $0 \leqq r < n$. They assert for the ordered integral domain $(\mathbb{Z}, +, \cdot, \mathbb{N})$ the following.

If $a, b \in [0, n[_{\mathbb{Z}}$ then

$$|a - b| \;\leqq\; \max(a,b), \quad \text{i.e.,} \tag{3.45}$$
$$-\max(a,b) \;\leqq\; a - b \;\leqq\; \max(a,b), \tag{3.46}$$
$$-n \;<\; a - b \;<\; n. \tag{3.47}$$

[1] i.e., $B = f(A)$