Math 330 Section 6 - Fall 2020 - Homework 14

Published: Monday, November 16, 2020 *Running total:* 50 *points*
Last submission: Monday, November 30, 2020 (after Thanksgiving)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - 7.1 (skip the remainder of ch.7), ch.8 - 13, appendix A

MF lecture notes:
ch.2 - 3, ch.5 - 13.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, November 16:

NONE

Reading assignment 2 - due: Wednesday, November 18:

a. Carefully read the remainder of MF ch.13. Draw plenty of pictures for the case of $X = \mathbb{R}^2$

Reading assignment 3 - due Friday, November 20:

a. Carefully read MF ch.14.1. Go back to ch.3.5 and ch.5.1 to reacquaint yourself with the definition of maxima and POsets. Be sure to understand the difference between maxima and maximal elements as their generalization!
b. Re–read the last part of MF ch.11.2.1, starting at the definition of linear independence.
c. Carefully read ch.14.2 and 14.3. Note that today's reading assignments for ch.14 cover barely more than 4 pages!

Written assignments are on the next page.

Written assignment 1 (2 points):
Prove part **d** of prop. 12.27 (Closure of a set as a hull operator): Let A and B be subsets of a topological space $(X, \mathfrak{U})$. Then

a. $\bar{A} \cup \bar{B} \subseteq \overline{A \cup B}$.

b. $\overline{A \cup B} \subseteq \bar{A} \cup \bar{B}$.

One point each for **(a)** and **(b)**!

Written assignment 2 (2 points):

Let $X := \mathbb{R}$ equipped with the standard Euclidean metric $d(x, x') = |x - x'|$. Let $f_n : \mathbb{R} \to \mathbb{R}$ be the following sequence of functions:

$$f_n(x) := \begin{cases} 0 & \text{if } |x| > \frac{1}{n}, \\ nx + 1 & \text{if } \frac{-1}{n} \leqq x \leqq 0, \\ -nx + 1 & \text{if } 0 \leqq x \leqq \frac{1}{n}, \end{cases}$$

i.e., the point $\big(x, f_n(x)\big)$ is on the straight line between $(-\frac{1}{n}, 0)$ and $(0, 1)$ for $\frac{-1}{n} \leqq x \leqq 0$, it is on the straight line between $(0, 1)$ and $(\frac{1}{n}, 0)$ for $0 \leqq x \leqq \frac{-1}{n}$, and it is on the x–axis for all other x. Draw a picture! Let $f(x) := 0$ for $x \neq 0$ and $f(0) := 1$.

a. Prove that f_n converges pointwise to f on $\mathbb{R}$.
b. Prove that f_n does not converge uniformly to f on $\mathbb{R}$. $\square$

You may use without proof that each of the functions f_n is continuous on $\mathbb{R}$.

One point each for (a) and (b)!

Hint for part **(a)**: For fixed $x \neq 0$, what happens eventually, i.e., for big enough n? Example (NOT legit as a proof): If x = 0.01, what happens if $n > 1000$? Thus $\lim\limits_{n \to \infty} f_n(0.01)$ = WHAT?