

Math 330 Section 6 - Fall 2020 - Homework 15

Published: Friday, November 18, 2020
Last submission: Wednesday, December 9, 2020

Running total: 52 points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:

ch.1 - 7.1 (skip the remainder of ch.7), ch.8 - 13, appendix A

MF lecture notes:

ch.2 - 3, ch.5 - 14.3

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:

Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment - Thanksgiving Week:

- Carefully read (the optional) MF ch.14.4 (less than a page and a half).
- Carefully read (the optional) MF ch.14.5 but SKIP lemma 14.3 and lemma 14.4. They are very technical and they will only appear in the proof of the Hahn-Banach theorem.
- Carefully read (the optional) MF ch.15, including the introductions, through introduction 15.3 at the start of ch.15.2. You may recall that we talked at the very end of the Fri 11/20 lecture about the uniform convergence $B_n^f(\cdot) \xrightarrow{uc} f(\cdot)$ on $[0, 1]$ as $n \rightarrow \infty$ when $f(x) = 1$, $f(x) = x$, or $f(x) = x^2$.

The written assignments of this homework set require you to work with sequence compact metric spaces. Here is the definition from ch.13.4 (Sequence Compactness):

We say that the metric space (X, d) is **sequence compact** if it has the following property: Given any sequence (x_n) of elements of X , there exists $x \in X$ and a subsequence $(x_{n_j})_j$ of $(x_n)_n$ such that $\lim_{j \rightarrow \infty} x_{n_j} = x$.

Written assignment 1:

Let X be the open unit interval $]0, 1[$, equipped with the Euclidean metric $d(x, x') = |x' - x|$. Prove that X is **not** sequence compact by finding a sequence $x_n \in]0, 1[$ which does not possess a limit in $]0, 1[$.

Written assignment 2:

Let X be a (abstract) finite and nonempty set, equipped with the discrete metric. Prove that X is sequence compact.