Math 330 Section 4 - Fall 2021 - Homework 01

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Update August 25, 2021

Removed the "Optional, but strongly recommended" part of Monday's reading assignment: You only need the reading for Fri 9/27 to work on the written assignments!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date: None so far since this is your first homework assignment.

New reading assignments:

Reading assignment 1 - due Friday, August 27(!):

- **a.** Review MF ch.2.1 (Sets and Basic Set Operations) and ch.2.2 (Numbers). Those chapters are meant for self–study since most if not all of the material found there should be familiar to you. You need to be comfortable with the differences between natural numbers, integers, and rational numbers. Note that this material is considered general knowledge for anyone who has studied at least one semester of calculus, a prerequisite for this course. I will skip most of ch.2.1 and 2.2 in class.
- **b.** Read carefully MF ch.2.3 (A First Look at Functions and Sequences). Pay particular attention to the motivational note 2.1!
- **c.** Read carefully MF ch.3.1 (Semigroups and Groups) through theorem 3.3.

Reading assignment 2 - due Monday, August 30:

- **a.** Read carefully the remainder of MF ch.3.1.
- **b.** Read carefully MF ch.3.2 (Commutative Rings and Integral Domains).

Reading assignment 3 - due Wednesday, September 1:

- **a.** Read carefully MF ch.3.3 (Arithmetic in Integral Domains).
- **b.** Read carefully B/G ch.1 (Integers). Study the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 really describe what you know as "integers" or are they more specific or more general than that set of numbers?

Reading assignment 4 - due Friday, September 3:

- **a.** Read carefully MF ch.3.4 (Order Relations in Integral Domains).
- **b.** Read carefully B/G ch.2.1 (Natural Numbers) and ch.2.2 (Ordering the Integers). Study again the connections to what you have already learned from MF ch.3! Do B/G axioms 1.1 – 1.5 PLUS ax.2.1 really describe what you know as "integers" or are they more specific or more general than that set of numbers?
- **c.** Optional, but highly recommended if you lack familiarity with basic set operations and functions with arbitrary domain/codomain: Read the following from the B/K (Bryant/Kirby) lecture notes:
  - ch.1.1 (Introduction to sets)
  - ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Written assignments:

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

Hint for ALL written assignments:
Before you start this homework set you must do reading assignment 1. If you understand MF ch.3.1 through Theorem 3.3 then you have everything you need to work on those exercises.

Written assignment 1: Prove MF Exercise 3.2:

Let $(G, \diamond)$ be a commutative group with neutral element e. Let $g, h_1, h_2 \in G$ such that

$$g \diamond h_1 \ = e \qquad \text{and} \qquad g \diamond h_2 \ = e \, .$$

Prove that $h_1 = h_2$.

Hints for assignments #2 and #3:
a. You do not have commutativity as a tool and that is a good thing: the variables appear in the same left–to–right order on both sides!
b. Obviously you'll have to utilize associativity of $\diamond$ to prove #2 and #3. Tell me me what you plug in for s, t, u in formula (3.1)!

Written assignment 2: This is MF Exercise 3.3 **(a)**: Let $(S, \diamond)$ be a semigroup. Let $a, b, c, d \in S$. Prove that

$$a \ \diamond \ \big(b \diamond (c \diamond d)\big) \ = \ (a \diamond b) \diamond (c \diamond d) \, .$$

Written assignment 3: This is MF Exercise 3.3 **(b)**: Let $(S, \diamond)$ be a semigroup. Let $a, b, c, d \in S$. Prove that

$$\big(a \diamond (b \diamond c)\big) \ \diamond \ d \ = \ (a \diamond b) \ \diamond \ (c \diamond d) \, .$$

Written assignment 4: This is MF Exercise 3.4: Let $(S, \diamond)$ be a commutative semigroup, i.e., S is a semigroup which satisfies $s \diamond t \ = \ t \diamond s$ for all $s, t \in S$. Let $a, b, c \in S$. Prove that

$$a \diamond (b \diamond c) \ = \ c \diamond (a \diamond b)$$

Do not forget written assignment zero:

Send an email by Friday, August 28, that
a. lists your math background, including for the current semester(!),
b. acknowledges that you have read the syllabus posted on the course website and/or on Blackboard,
c. tells me why you chose to take this course