Math 330 Section 4 - Fall 2021 - Homework 02

Published: Thursday, September 2, 2021 Running total: 14 points

Last submission: Wednesday, September 15, 2021

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date:

B/G (Beck/Geoghegan) Textbook: ch.1, ch.2.1 - 2.2

MF lecture notes:

ch.2.1 - 2.3, ch.3.1 - 3.4

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments: NONE!

You will get your next reading assignments in homework 3.

Written assignments:

Partial credit will be given. You can earn as many as 10 points!

Helpful hints:

- **a.** No matter what A stands for, it is never true that $A = \{A\}$. Not even if $A = \emptyset$ (the empty set): $\{A\} = \{\emptyset\}$ is a set which contains an element (exactly one): The empty set! Because $\{\emptyset\}$ is not empty it follows that $\{\emptyset\} \neq \emptyset$. By the way: It is true that $\emptyset \subseteq \{\emptyset\}$!
- **b.** No matter what A stands for, it is never true that $A \in A$. Again, not even if $A = \emptyset$ (the empty set): The empty set contains nothing at all; in particular, it does not contain any set; in particular, it does not contain the set that has no elements, i.e., the empty set. Thus $\emptyset \notin \emptyset$.
- **c. CAREFUL HERE**: It is possible to have both $a \in U$ and $\{a\} \in U$. Matter of fact, the first assignment of this homework contains such an example.

Note the following:

A. In the MF doc refer to def.2.10 for the preliminary definition of the size of a set S: If S is finite then |S| is the number of elements of S, otherwise $|S| = \infty$.

B. Refer to MF doc def.2.22 (Preliminary definition: cartesian product) for the definition of $X \times Y$.

Written assignment 1: Let $A = \{u, w, \{w\}, \{u, w\}\}$. true or false?

a.
$$\{w\} \subseteq A$$

b.
$$\{\{w\}\}\subseteq A$$

c.
$$\{u\} \subseteq A$$

d.
$$u \subseteq A$$

e.
$$\{w\} \in A$$

f.
$$\{ \{ w \} \} \in A$$

g.
$$\{u\} \in A$$

h.
$$u \in A$$

Written assignment 2: Find the size of each of the following sets:

a.
$$A = \{6, \{6\}, \{-6\}\}$$

b.
$$F = \{\sin(x) : x \in \mathbb{R}\}$$

c.
$$D = \{2, 3, 4, 3, 2\}$$

d.
$$C = \{5z - 3z^2 : z \in \mathbb{Z}\}$$

e.
$$B = \{4, \{4\}, \pi, \{4\}, \{\pi\}, \{4, \pi\}\}$$

f.
$$E = \{(-1)^k : k \in \mathbb{Z}\}$$

Written assignment 3:

Let $U = \{a, \{b\}\}$ and $V = \{a, b, \{a\}, \{a, b\}\}$. true or false?

a.
$$\{b\} \in U \cap V$$

b.
$$\{b\} \in U \setminus V$$

c.
$$a \in U \cap V$$

d.
$$a \in U \setminus V$$

$$e. \quad \{b\} \in U \cup V$$

f.
$$\{b\} \in U\Delta V$$

g.
$$a \in U \cup V$$

h.
$$a \in U\Delta V$$

Written assignment 4: Let $G = \{3, 4\}$, and let $H = \{u, v, w\}$.

a. What is
$$G \times H$$
?

b. What is
$$H \times G$$
?

c. What is
$$|G \times H|$$
?

d. What is
$$|H \times G|$$
?

e. Is
$$(4, u) \in G \times H$$
?
f. Is $(4, u) \in H \times G$?

g. Is
$$w \cdot 3 \in G \times H$$
?

h. Is
$$w \cdot 3 \in H \times G$$
?

Written assignment 5: Let $X = \{4\}$.

a. What is
$$2^X$$
?

b. What is
$$2^{(2^X)}$$
?