

Math 330 Section 4 - Fall 2021 - Homework 07

Published: Thursday, September 23, 2021
Last submission: Friday, October 8, 2021

Running total: 32 points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1, ch.2.1 - 2.2, ch.3

MF lecture notes:
ch.2-3, skim ch.4, ch.5-6.2

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due: Monday, September 27:

- a. Read carefully MF ch.6.3-6.4.
- b. Skim the optional MF ch.6.5 (Bernstein Polynomials).
- a. Read carefully B/G Ch.2.3 on induction. This material was covered in MF ch.6.1.

Reading assignment 2 - due: Wednesday, September 29:

- a. Read carefully MF ch.6.6-6.10.

Reading assignment 3 - due Friday, October 1:

- a. Read carefully the remainder of B/G Ch.2 and B/G ch.4. All of this material was covered in MF ch.6.1-6.8
- b. Read carefully B/G ch.6.1-6.3. All of this material was covered in MF ch.5.1 and MF ch.6.9-6.10.

Written assignments are on the next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

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Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$ according to MF def.6.11 in ch.6.6 (Divisibility) or the definitions that follow B/G prop.1.14.

Written assignment 2:

Prove MF Prop. 6.7a by induction on p : Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^p x_j = \sum_{j=m}^n x_j \oplus \sum_{j=n+1}^p x_j.$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p ? If $m = -4$ and $n = 8$, how would you choose p ? For general $m \leq n$, how would you choose p ?

Written assignment 3:

Let $x_0 = 8$, $x_1 = 16$, $x_{n+1} = 6x_{n-1} - x_n$ for $n \in \mathbb{N}$.

Prove that $x_n = 2^{n+3}$ for every integer $n \geq 0$.

Hint: Use strong induction.