

Math 330 Section 4 - Fall 2021 - Homework 12

Published: Thursday, October 28, 2021
Last submission: Friday, November 12, 2021

Running total: 47 points

Status - previously assigned reading Assignments:

B/G (Beck/Geoghegan) Textbook:
ch.1-7 (until Theorem 7.17), ch.8-11.3

MF lecture notes:
ch.2-3, ch.4 (skim), ch.5-8, ch.9.1-9.7

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due: Monday, November 1:

- Read carefully MF ch.9.8 until before Proposition 9.45 and skim the optional remainder.
- Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup).
- Skim the optional MF ch.9.10. The stronger students are encouraged to look closely at this very short chapter (less than a full page).

Reading assignment 2 - due: Wednesday, November 3:

- Read carefully MF ch.10. This is a very hard chapter, but note that the lengthy proof of Lemma 10.1. will be marked as optional in the forthcoming next edition of the MF lecture notes.

Reading assignment 3 - due Friday, November 5:

- Read carefully B/G ch.13.1. The material of this chapter has been covered in MF ch.7 and ch.10.

Written assignments:

Written assignment 1: Prove Proposition 9.2: Fields are integral domains.

Hint: Use Proposition 9.1 to show that there are no zero divisors.

Written assignment 2: Prove Theorem 9.2:

Let F be an ordered field and let $a, b \in F$ such that $a < b$. Then $a < \frac{a+b}{2} < b$.

Written assignment 3: Prove formula (9.14) of prop.9.11: Let X be a nonempty set and $\varphi, \psi : X \rightarrow \mathbb{R}$. Let $A \subseteq X$. Then

$$\inf\{\varphi(x) + \psi(x) : x \in A\} \geq \inf\{\varphi(y) : y \in A\} + \inf\{\psi(z) : z \in A\}.$$

Do this proof without applying formula (9.13) to $-\varphi$ and ψ .

Big hint: Examine the proof of formula (9.13) and follow it as closely as possible!