Math 330 Section 4 - Fall 2021 - Homework 13

Published: Tuesday, November 2, 2021 *Running total:* 49 *points*
Last submission: Friday, November 19, 2021

Status - previously assigned reading Assignments:

B/G (Beck/Geoghegan) Textbook:
ch.1-7 (until Theorem 7.17), ch.8-11.3, ch.13.1

MF lecture notes:
ch.2-3, ch.4 (skim), ch.5-10

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due: Monday, November 8:

a. Prepare for your midterm!

Reading assignment 2 - due: Wednesday, November 10:

a. Read MF ch.11.1 and ch.11.2.1. This should be a quick read since all of you have taken or are taking a linear algebra course. pay particular attention to example 11.11 (Vector spaces of real–valued functions) since we will work extensively with the function spaces $\mathscr{F}(X,\mathbb{R})$, $\mathscr{B}(X,\mathbb{R})$, $\mathscr{C}(X,\mathbb{R})$.
b. Read carefully MF ch.11.2.2 until before Proposition 11.14.

Reading assignment 3 - due Friday, November 12:

a. Read carefully the remainder of MF ch.11.2.2.
b. Skim MF ch.11.3 (the remainder of ch.11).

Written assignments are on the next page.

Written assignments:

Written assignment 1: Prove MF prop.9.18**(b)**: If y_n is a sequence of real numbers that is nonincreasing, i.e., $y_n \geqq y_{n+1}$ for all n, and bounded below, then $\lim\limits_{n\to\infty} y_n$ exists and coincides with $\inf\{y_n : n \in \mathbb{N}\}$.

Do the proof by modifying the proof of prop.9.18**(a)**. You are **NOT ALLOWED** to apply prop.9.18**(a)** to the sequence $x_n := -y_n$!

Written assignment 2:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$. You MUST work with ε-δ continuity (thm.9.7) **NOT WITH SEQUENCE CONTINUITY**, and you cannot use any "advanced" knowledge such as the product of continuous functions being continuous, etc.

Special instructions for assignment 1: Turn in your scratchpaper where you solve for δ (see the hints below).

Hints:

- **a.** What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
- **b.** $x^2 - 1 = (x+1)(x-1)$.
- **c.** Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then "solving for δ" That part should not be in your official proof.
- **c1.** Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $0 < \varepsilon < 1$. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\ x^2 - 1|$, $|x+1|$, $|x-1|$? if $0 < \delta < 1$?
- **c2.** Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given? You'll get the answer by "solving for δ".
- **c3.** All of the above was done under the assumption that $\delta < 1$. Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$
- **d.** Only now you are ready to construct an acceptable proof: Let $\varepsilon > 0, \delta := \dots$, and $\delta' := \min(\delta, 1)$. Then ....