Math 330 Section 4 - Fall 2021 - Homework 17

Published: Tuesday, November 30, 2021 *Running total: 62 points*
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Status - previously assigned reading Assignments:

B/G (Beck/Geoghegan) Textbook:
ch.1-7 (until Theorem 7.17), ch.8-13

MF lecture notes:
ch.2-3, ch.4 (skim), ch.5-ch.14.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

No required reading for this last week. Rather some recommendations for your final exam prep.

Reading assignment 1 - due: Monday, December 6 - Final exam prep:

- **a.** Review your quizzes and midterms.
- **b.** Review in parallel convergence and continuity in ch.9 and ch.12. Understand the connection between convergent and Cauchy sequences, continuous and bounded functions.
- **d.** Review the notions of finite, countable, countably infinite, uncountable: MF ch.7, ch.10, ch.9.7.
- **c.** Understand Remark 12.15 (Hierarchy of topological spaces).

Reading assignment 2 - due: Wednesday, December 8 - Final exam prep:

- **a.** Review B/G ch.3 on logic
- **b.** Review fully worked proofs by induction, e.g., in MF ch.2, ch.6, ch.20 and B/G ch.2 and ch.4.
- **c.** Review sets and functions: MF ch.2, ch.5, ch.8
- **c.** Understand that the metric space $(\mathbb{R}, |\cdot|)$ is complete whereas $(\mathbb{Q}, |\cdot|)$ is not.

Reading assignment 3 - due Friday, December 10:

- **a.** OPTIONAL: Read MF ch.14.5 but SKIP Lemma 14.3 and Lemma 14.4. Those are very technical and they are used only in the proof of the Hahn-Banach theorem. Try to understand the definition of sublinearily well enough to follow the proof of Theorem Theorem 14.4 (Hahn–Banach Extension Theorem):
- **b.** OPTIONAL: Read MF ch.14.6. It gives you some understanding of concave–up and concave–down without the notion of differentiability.

Written assignments on next page.

The first two written assignments of this homework set require you to work with sequence compact metric spaces. You can solve those problems with your knowledge of convergence in metric spaces but you also need to work with the following definition from ch.13.4 (Sequence Compactness):

We say that the metric space (X, d) is **sequence compact** if it has the following property: Given any sequence (x_n) of elements of X, there exists $x \in X$ and a subsequence $(x_{n_j})_j$ of $(x_n)_n$ such that $\lim\limits_{j\to\infty} x_{n_j} = x$.

Written assignment 1:
Given is a metric space (X, d) and two functions $f, g : X \to \mathbb{R}$ which are continuous at $x_0 \in X$. Assume that $g(x_0) \neq 0$. Prove that the quotient $x \to \frac{f(x)}{g(x)}$ is continuous at x_0.

Hint: Work with sequence continuity and prop.9.17 (Rules of arithmetic for limits): What will be the sequences of real numbers you will apply this proposition to?