Math 330 Section 5 - Spring 2022 - Homework 02

Published: Wednesday, November 24, 2021 *Running total:* 14 *points*
Last submission: Wednesday, February 9, 2022

Last submission date has been changed from Feb 2 to Feb 9

Previously assigned reading:

MF lecture notes:
ch.2.1 - 2.3, ch.3.1 - 3.4 until Def.3.12 (Absolute value)

B/G (Beck/Geoghegan) Textbook (Optional):
ch.1, ch.2.1 - 2.2

B/K lecture notes (Optional):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments: NONE!

You will get your next reading assignments in homework 3.

Written assignments:

Partial credit will be given. You can earn as many as 10 points!

Helpful hints:

a. No matter what A stands for, it is never true that $A = \{A\}$. Not even if $A = \emptyset$ (the empty set): $\{A\} = \{\emptyset\}$ is a set which contains an element (exactly one): The empty set! Because $\{\emptyset\}$ is not empty it follows that $\{\emptyset\} \neq \emptyset$. By the way: It is true that $\emptyset \subseteq \{\emptyset\}$!

b. No matter what A stands for, it is never true that $A \in A$. Again, not even if $A = \emptyset$ (the empty set): The empty set contains nothing at all; in particular, it does not contain any set; in particular, it does not contain the set that has no elements, i.e., the empty set. Thus $\emptyset \notin \emptyset$.

c. **CAREFUL HERE**: It is possible to have both $a \in U$ and $\{a\} \in U$. Matter of fact, the first assignment of this homework contains such an example.

Note the following:

A. In the MF doc refer to def.2.10 for the preliminary definition of the size of a set S:
If S is finite then $|S|$ is the number of elements of S, otherwise $|S| = \infty$.

B. Refer to MF doc def.2.22 (Preliminary definition: cartesian product) for the definition of $X \times Y$.

Written assignment 1: Let $A = \{u, w, \{w\}, \{u, w\}\}$. **true** or **false**?

- **a.** $\{w\} \subseteq A$
- **b.** $\{\{w\}\} \subseteq A$
- **c.** $\{u\} \subseteq A$
- **d.** $u \subseteq A$
- **e.** $\{w\} \in A$
- **f.** $\{\{w\}\} \in A$
- **g.** $\{u\} \in A$
- **h.** $u \in A$

Written assignment 2: Find the size of each of the following sets:

- **a.** $A = \{6, \{6\}, \{-6\}\}$
- **b.** $F = \{\sin(x) : x \in \mathbb{R}\}$
- **c.** $D = \{2, 3, 4, 3, 2\}$
- **d.** $C = \{5z - 3z^2 : z \in \mathbb{Z}\}$
- **e.** $B = \{4, \{4\}, \pi, \{4\}, \{\pi\}, \{4, \pi\}\}$
- **f.** $E = \{(-1)^k : k \in \mathbb{Z}\}$

Written assignment 3:

Let $U = \{a, \{b\}\}$ and $V = \{a, b, \{a\}, \{a, b\}\}$. **true** or **false**?

- **a.** $\{b\} \in U \cap V$
- **b.** $\{b\} \in U \setminus V$
- **c.** $a \in U \cap V$
- **d.** $a \in U \setminus V$
- **e.** $\{b\} \in U \cup V$
- **f.** $\{b\} \in U \Delta V$
- **g.** $a \in U \cup V$
- **h.** $a \in U \Delta V$

Written assignment 4: Let $G = \{3, 4\}$, and let $H = \{u, v, w\}$.

- **a.** What is $G \times H$?
- **b.** What is $H \times G$?
- **c.** What is $|G \times H|$?
- **d.** What is $|H \times G|$?
- **e.** Is $(4, u) \in G \times H$?
- **f.** Is $(4, u) \in H \times G$?
- **g.** Is $w \cdot 3 \in G \times H$?
- **h.** Is $w \cdot 3 \in H \times G$?

Written assignment 5: Let $X = \{4\}$.

- **a.** What is 2^X?
- **b.** What is $2^{\left(2^X\right)}$?

Remember that you are dealing with power sets, so the answers must be sets and NOT numbers!