Math 330 Section 5 - Spring 2022 - Homework 05

Published: Tuesday, February 15, 2022 *Running total: 25 points*
Last submission: Monday, February 28, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2.1 - 2.4, ch.3, ch.4 (skim), ch.5 - 6.2

B/G (Beck/Geoghegan) Textbook:
ch.1, ch.2.1 - 2.3, ch.3, ch.5

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

New reading assignments: None

Written assignments:

These written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

Written assignment 1:

Injectivity and Surjectivity

- Let $f : \mathbb{R} \longrightarrow [0, \infty[; \quad x \mapsto x^2$.
- Let $g : [0, \infty[\longrightarrow [0, \infty[; \quad x \mapsto x^2$.
  In other words, g is same function as f as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with **true** or **false**.

a. f is surjective **c.** g is surjective
b. f is injective **d.** g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2: Find $f : X \longrightarrow Y$ and $A \subseteq X$ such that $f(A^{\complement}) \neq f(A)^{\complement}$.

Hint: Use $f(x) = x^2$ and choose Y as a **one element only** set (which does not leave you a whole lot of choices for X). See MF example 5.19 with the "arrows diagram". Start this problem as follows: Let $X := \{......\},\ A := \{......\},\ Y := \{......\}$.

Written assignment 3: Let $f:]-10, 10[\longrightarrow \mathbb{R}; \quad x \mapsto x^2$.

a. what is the range of f? **b.** Is f injective? **c.** Is f surjective?

d. $f(\{1\} \cup [4,6]) \ =?$ **e.** $f([2,5]) \cap f([4,7]) =?$ **f.** $f^{-1}([4,25]) \cap f^{-1}([16,49]) =?$

Hint: For **d, e, f**, review examples 5.24–5.27.

Written assignment 4:

You have learned in MF ch.5 that
injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective.

The following illustrates that the reverse is not necessarily true.

Assume that $b_1 \neq b_2$. Find functions $f: \{a\} \to \{b_1, b_2\}$ and $g: \{b_1, b_2\} \to \{a\}$ which satisfy the following: The composition $h := g \circ f : \{a\}$ is bijective but it is **not true** that both f, g are injective, and it is also **not true** that both f, g are surjective. **You are NOT ALLOWED use any other sets (symbols) when doing this problem!**

Hint: There are not a whole lot of possibilities. Draw all possible candidates for f and g in arrow notation as you see in MF example 5.19. There are only very few choices!