Math 330 Section 5 - Spring 2022 - Homework 07

Published: Thursday, February 24, 2022 *Running total:* 32 *points*
Last submission: Friday, March 11, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2.1 - 2.4, ch.3, ch.4 (skim), ch.5 - 6.11 (skip 6.5)

B/G (Beck/Geoghegan) Textbook:
ch.1 - 6.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, February 28:

- **a.** Read carefully the remainder of MF ch.6.
- **b.** Read carefully B/G ch.6. You will find there some proofs missing from MF ch.6.10 and 6.11.

Reading assignment 2 - due: Wednesday, March 2:

- **a.** Read carefully MF ch.7.1 and ch.7.2.

Reading assignment 3 - due Friday, March 4:

- **a.** Read B/G ch.7.1.
- **b.** Prepare for the first midterm!

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:
Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$ according to MF def.6.11 in ch.6.6 (Divisibility) or the definitions that follow B/G prop.1.14.

Written assignment 2:
Prove MF Prop. 6.7**(a)** by induction on p: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain

$R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^{p} x_j \;=\; \sum_{j=m}^{n} x_j \;\oplus\; \sum_{j=n+1}^{p} x_j .$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p? If $m = -4$ and $n = 8$, how would you choose p? For general $m \leqq n$, how would you choose p?

Written assignment 3:
Let $x_0 = 8,\; x_1 = 16,\; x_{n+1} = 6x_{n-1} - x_n$ for $n \in \mathbb{N}$.

Prove that $x_n = 2^{n+3}$ for every integer $n \geq 0$.

Hint: Use strong induction.