Math 330 Section 5 - Spring 2022 - Homework 11

Published: Thursday, March 31, 2022 *Running total:* 44 *points*
Last submission: Tuesday, April 19, 2022 (after the break)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2.1 - 2.4, ch.3, ch.4 (skim), ch.5 - 8 (skip 6.5, skim 8.2, 8.4 after Prop.8.11, ch.8.5), ch.9.1 - 9.7

B/G (Beck/Geoghegan) Textbook:
ch.1 - 7.1, ch.8-9

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 9ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 4:

- **a.** Read carefully MF ch.9.8 until before Proposition 9.44 and skim the optional remainder.
- **b.** Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup). The stronger students are encouraged to skim the contents, in particular the last remark (new with ver 2022-03-31).
- **c.** Skim the optional MF ch.9.10. The stronger students are encouraged to look closely at this very short chapter (less than a full page).

Reading assignment 2 - due Wednesday, April 6:

- **a.** Read carefully MF ch.10.1 and ch.10.2. It is brief but contains many very important results concerning the size of certain subsets of $\mathbb{R}$.
- **b.** Skip the remainder of MF ch.10.

Reading assignment 3 - due Friday, April 8:

- **a.** Review B/G ch.13. You know all its material from MF ch.7 and ch.10.
- **b.** **Important if you have not taken/are not taking linear algebra:** Read MF ch.11.1. Be sure to memorize Definition 11.3 and Proposition 11.1 and to dig into the (parts given there of the) proof of that proposition. All of this should be a very easy read.

Written assignments are on p.2.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Prove Proposition 7.13: Every infinite set contains a proper subset that is countably infinite.

Written assignment 2: Prove the following part of De Morgan's Law:

Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha\in I}$. Then

$$\Big(\bigcap_\alpha A_\alpha\Big)^{\complement} \subseteq \bigcup_\alpha A_\alpha^{\complement} .$$

Written assignment 3: Prove formula (8.32):

If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, $U \subseteq X$, and $W \subseteq Z$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X .$$