Math 330 Section 5 - Spring 2022 - Homework 13

Published: Tuesday, April 12, 2022 *Running total:* 50 *points*
Last submission: Friday, April 29, 2022

Update April 14, 2022

Last submission date is April 29, NOT April 30

Chapter 12 references were **ERRONEOUS** and have been replaced with ch.11 references!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2.1 - 2.4, ch.3, ch.4 (skim), ch.5 - 8 (skip 6.5, skim 8.2, 8.4 after Prop.8.11, ch.8.5), ch.9 skim/skip after Prop.9.44), ch.10.1-10.2, ch.11-11.2.1

B/G (Beck/Geoghegan) Textbook:
ch.1 - 7.1, ch.8-11, ch.13

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 9ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 19 (optional):

- Review ch.10 and the beginning of B/G ch.13 (the part that deals with cardinality)

Reading assignment 2 - due Wednesday, April 20:

a. OUT OF SEQUENCE: Read carefully ch.14.1 and ch.14.3 but skip ch.14.2.
b. For the better students: Read ch.5.4 (Right Inverses and the Axiom of Choice).
c. Read carefully ch.11.2.2 (NOT ch.12.2.2) until before Definition 11.14.

Reading assignment 3 - due Friday, April 22:

a. Carefully read the remainder of MF ch.11.2.2 (NOT ch.12.2.2).
b. For the better students: Read the optional ch.11.2.3 (NOT ch.12.2.3) and notice how ingeneously the three properties of a norm are put to work.

Written assignments are on p.2.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$. You MUST work with ε-δ continuity (thm.9.7) **NOT WITH SEQUENCE CONTINUITY**, and you cannot use any "advanced" knowledge such as the product of continuous functions being continuous, etc.

This assignment is worth up to 3 points. Partial credit will be given and you can turn it in repeatedly.

Special instructions for assignment 1: Turn in your scratchpaper where you solve for δ (see the hints below).

Hints:

- **a.** What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
- **b.** $x^2 - 1 = (x+1)(x-1)$.
- **c.** Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then "solving for δ" That part should not be in your official proof.
- **c1.** Only small neighborhoods matter (see Proposition 9.24 at the end of the chapter on convergence and continuity.): Given $0 < \varepsilon < 1$ try to find δ that works for such ε. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\ x^2 - 1|$, $|x+1|$, $|x-1|$ if $0 < \delta < 1$?
- **c2.** Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given? You'll get the answer by "solving $|f(x) - f(x_0)| \leq 3\delta$ for δ".
- **c3.** All of the above was done under the assumption that $\delta < 1$. Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$
- **d.** Only now you are ready to construct an acceptable proof: Let $\varepsilon > 0, \delta := \ldots$, and $\delta' := \min(\delta, 1)$. Then ....

Written assignment 2: Prove MF Thm. 9.8: If $m \in [0, \infty[_{\mathbb{Z}}$ is not a perfect square then $\sqrt{m}$ is irrational.

Hint: Work with lowest term representations. See the proof that $\sqrt{2}$ is irrational.

No partial credit for this one!