Math 330 Section 5 - Fall 2022 - Homework 08

Published: Tuesday, September 20, 2022 *Running total:* 32 *points*
Last submission: Friday, October 7, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

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MF lecture notes:
ch.2 – ch.3; skim ch.4; ch.5.1 – ch.5.2; ch.6

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.6; skim ch.7

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 3:

a. Read carefully MF ch.7.1 - 7.2.

Reading assignment 2 - due Wednesday, October 5:

a. Read carefully the remainder of MF ch.7.
b. Read carefully MF ch.8.1 and 8.3. Skim MF ch.8.2.

Reading assignment 3 - due Friday, October 7:

a. Read carefully MF ch.8.4 until until before Proposition 8.11 and skim the remainder of this chapter..

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:
Prove B/G Prop. 4.7(i) by induction: Let $k \in \mathbb{N}$. Then there exists $j \in \mathbb{Z}$ such that $5^{2k} - 1 = 24j$. In other words, $24 \mid (5^{2k} - 1)$.

Written assignment 2:
Prove MF Prop. 6.7**(a)** by induction on p: Let $(x_j)_{j\in\mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^{p} x_j \ = \ \sum_{j=m}^{n} x_j \ \oplus \ \sum_{j=n+1}^{p} x_j .$$

Hints: Think carefully about the base case: If $m = 5$ and $n = 8$, how would you choose p? If $m = -4$ and $n = 8$, how would you choose p? For general $m \leqq n$, how would you choose p?

Written assignment 3:
Let $x_0 = 8,\ x_1 = 16,\ x_{n+1} = 6x_{n-1} - x_n$ for $n \in \mathbb{N}$.

Prove that $x_n = 2^{n+3}$ for every integer $n \geq 0$.

Hint: Use strong induction.