Math 330 Section 5 - Fall 2022 - Homework 11

Published: Tuesday, October 18, 2022 *Running total:* 40 *points*
Last submission: Friday, November 4, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2 – ch.3; skim ch.4; ch.5.1 – ch.5.2; ch.6 - 9.5

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.6; skim ch.7; ch.8, ch.10.1 – 10.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 24:

- **a.** Read carefully MF ch.9.6, 9.7.
- **b.** Skim the following B/G chapters: B/G 10.5 and ch.11.1 – 11.2

Reading assignment 2 - due: Wednesday, October 26:

- **a.** Read carefully MF ch.9.8 until before Proposition 9.43 and skim the optional remainder.
- **b.** Skip the optional MF ch.9.9 (Sequences of Sets and Indicator functions and their liminf and limsup). The stronger students are encouraged to skim the contents, in particular the last remark.
- **c.** Skim the optional MF ch.9.10. The stronger students are encouraged to look closely at this very short chapter (less than a full page).

Reading assignment 3 - due Friday, October 28:

- **a.** Read carefully MF ch.10.1 and ch.10.2. It is brief but contains many very important results concerning the size of certain subsets of $\mathbb{R}$.
- **b.** Skip the remainder of MF ch.10.

Written assignments are on the next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Prove Proposition 7.13: Every infinite set contains a proper subset that is countably infinite.

Written assignment 2: Prove the following part of De Morgan's Law:

Let there be a universal set Ω which contains all elements of an indexed family of sets $(A_\alpha)_{\alpha \in I}$. Then

$$\big(\bigcap_\alpha A_\alpha\big)^{\complement} \subseteq \bigcup_\alpha A_\alpha^{\complement}.$$

Written assignment 3: Prove formula (8.32):

If X, Y, Z be arbitrary, nonempty sets and $f : X \to Y$, $g : Y \to Z$, $U \subseteq X$, and $W \subseteq Z$, then

$$(g \circ f)(U) \subseteq g\big(f(U)\big) \text{ for all } U \subseteq X.$$