Math 330 Section 5 - Fall 2022 - Homework 12

Published: Tuesday, October 15, 2022 *Running total: 42 points*
Last submission: Friday, November 11, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2 – ch.3; skim ch.4; ch.5.1 – ch.5.2; ch.6 - 10

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.6; skim ch.7; ch.8, ch.10.1 – 11.2; skip ch.11.3

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 31:

- **a.** Read carefully MF ch.11.1. This should be an easy read even if you did not attend a linear algebra or Math 323.
- **b.** Read carefully MF ch.11.2.1 through Example 11.11 and skim the remainder. Be sure to attend my linear algebra tutorial, which I'll give either on Sat 11/29 or Sun 11/30, if you did not or do not currently attend a linear algebra lecture.

Reading assignment 2 - due: Wednesday, November 2:

- **a.** Read carefully MF ch.11.2.2. (NOT "the remainder of MF ch.11.2.2" as originally stated!)
- **b.** Optional: The stronger students should look at ch.12.2.3.

Reading assignment 3 - due Friday, November 4:

- **a.** Read carefully MF ch.12.1 and ch.12.2. It is crucial that you become familiar with the examples given there
- **b.** Read carefully MF ch.12.3. What does an open set in $\mathbb{R}$ with $d(x, y) = |y - x|$ look like?

Be sure to read pages 2 and 3!

Supplementary instructions for reading MF ch.12:

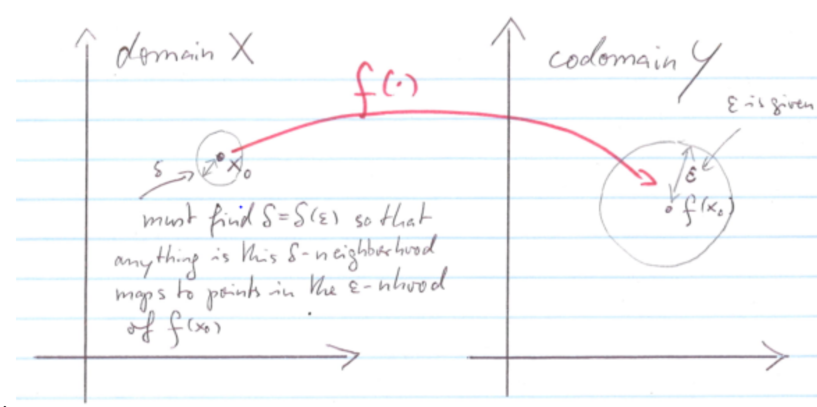
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

- open sets and neighborhoods (ch.12.1.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
- metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \uplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.12.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



Written assignments on page 3

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Prove formula (9.14) of prop.9.11: Let X be a nonempty set and $\varphi, \psi : X \to \mathbb{R}$. Let $\emptyset \neq A \subseteq X$. Then

$$\inf\{\varphi(x) + \psi(x) : x \in A\} \geqq \inf\{\varphi(y) : y \in A\} + \inf\{\psi(z) : z \in A\}.$$

Do the proof by modifying the proof of formula (9.13). Follow that proof as closely as possible!
You are **NOT ALLOWED** to apply formula (9.13) to $-\varphi$ and $-\psi$.

Written assignment 2: Prove MF prop.9.18**(b)**: If y_n is a sequence of real numbers that is nonincreasing, i.e., $y_n \geqq y_{n+1}$ for all n, and bounded below, then $\lim\limits_{n\to\infty} y_n$ exists and coincides with $\inf\{y_n : n \in \mathbb{N}\}$.

Do the proof by modifying the proof of prop.9.18**(a)**. You are **NOT ALLOWED** to apply prop.9.18**(a)** to the sequence $x_n := -y_n$!