Math 330 Section 5 - Fall 2022 - Homework 16

Published: Sunday, November 20, 2022 *Running total:* 59 *points*
Last submission: Friday, December 9, 2022

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

MF lecture notes:
ch.2 – ch.3; skim ch.4; ch.5.1 – ch.5.2; ch.6 - 14.1, Prop.14.1; ch.15.1 – 15.3

B/G (Beck/Geoghegan) Textbook:
ch.1 – ch.6; skim ch.7; ch.8, ch.10.1 – 11.2; skip ch.11.3; ch.13.1 – 13.4

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, November 28:

- **a.** Carefully read the remainder of MF ch.14.1 Note how similar the proofs of Proposition 14.2 and Theorem 14.2 are!
- **b.** Carefully read MF ch.14.2 Very short and very important!

Reading assignment 2 - due: Wednesday, November 30:

- Carefully read MF ch.14.3.

Reading assignment 3 - due Friday, December 2:

- **a.** Carefully read the remainder of MF ch.14.
- **b.** Take a closer look at MF ch.15.2.

Written assignment 1:
Prove Proposition 12.28**(d)**: Let A and B be subsets of a topological space $(X, \mathfrak{U})$. Then $\overline{A \cup B} = \bar{A} \cup \bar{B}$. Hint: OK to use parts **(b)** and **(c)** of that proposition.

Written assignment 2:
Theorem 13.3**(d)**: Given is a metric space (X, d) and two functions $f, g : X \to \mathbb{R}$ which are continuous at $x_0 \in X$. Assume that $g(x_0) \neq 0$. Prove that the quotient $x \to \frac{f(x)}{g(x)}$ is continuous at x_0.

Written assignments continued on the next page.

Hint: Work with sequence continuity and prop.9.17 (Rules of arithmetic for limits): Why does it help to know that $\lim_{n\to\infty}(1/a_n) = 1/\lim_{n\to\infty} a_n$? What the sequence $(a_n)_n$ you will apply this proposition to?

Written assignment 3:
Let X be the open unit interval $]0,1[$, equipped with the Euclidean metric $d(x,x') = |x' - x|$. Prove that X is **not** sequence compact by finding a sequence $x_n \in]0,1[$ for which no subsequence possesses a limit in]0,1[.

Written assignment 4:
Let X be a (abstract) finite and nonempty set, equipped with the discrete metric. Prove that X is sequence compact.