Math 330 Section 5 - Fall 2023 - Homework 02

New reading assignments NONE:

Written assignments:

- The material for all assignments, including the size of a set, is covered in MF ch.2.1 (Sets and Basic Set Operations) and MF ch.2.5 (Cartesian Products).
- This set is worth ⟨ **ten points!** ⟩ (probably translates to more than 70 grade points)!
- Graded ONLY ONCE, but partial credit will be given.

Clarification:

---
a. **Correct**: No matter what A stands for, it is never true that $A = \{A\}$. Not even if $A = \emptyset$ (the empty set): $\{\emptyset\}$ is a set: it is of the form $\{......\}$. But $\{\emptyset\}$ contains an element (exactly one): The empty set! So $\{\emptyset\} \neq \emptyset$. By the way: It is true that $\emptyset \subseteq \{\emptyset\}$!

b. **Correct**: No matter what A stands for, it is never true that $A \in A$. Again, not even if $A = \emptyset$ (the empty set): The empty set contains nothing at all; in particular, it does not contain any set; in particular, it does not contain the set that has no elements, i.e., the empty set.

c. **CAREFUL HERE**: If I told anyone of you that it is impossible to have both $a \in U$ and $\{a\} \in U$ then I made a mistake. Matter of fact, the first assignment of this homework has an example that this is possible.

Written assignment 1:

Let $S = \{3, 5, \{3, 5\}, \{5\} \}$. True or false?

a. $\{5\} \subseteq S$ **c.** $\{ \{5\} \} \subseteq S$ **e.** $\{3\} \subseteq S$ **g.** $3 \subseteq S$
b. $\{5\} \in S$ **d.** $\{ \{5\} \} \in S$ **f.** $\{3\} \in S$ **h.** $3 \in S$

Written assignment 2:

Find the size of each of the following sets:

a. $A = \{x, \{x\}, y, \{x\}, \{x, y\}\}$ **c.** $C = \{j, k, j, k, j\}$ **e.** $E = \{e^x : x \in \mathbb{R}\}$

b. $B = \{a, \{a\}, \{b\}\}$ **d.** $D = \{4q^2 : q \in \mathbb{Z}\}$ **f.** $F = \{(-1)^m : m \in \mathbb{Z}\}$

Written assignment 3:

Let $X = \{x, y, \{x\}, \{x, y\}\}$ and $Y = \{x, \{y\}\}$. True or false?

a. $x \in X \cap Y$ **c.** $x \in X \cup Y$ **e.** $x \in X \setminus Y$ **g.** $x \in X \Delta Y$

b. $\{y\} \in X \cap Y$ **d.** $\{y\} \in X \cup Y$ **f.** $\{y\} \in X \setminus Y$ **h.** $\{y\} \in X \Delta Y$

Written assignment 4:

Let $X = \{x, y\}$ and let $Y = \{1, 2, 3\}$.

a. What is $X \times Y$?   **c.** What is $\text{card}(X \times Y)$?   **e.** Is $(x, 3) \in X \times Y$?   **g.** Is $3 \cdot x \in X \times Y$?

b. What is $Y \times X$?   **d.** What is $\text{card}(Y \times X)$?   **f.** Is $(x, 3) \in Y \times X$?   **h.** Is $2 \cdot y \in Y \times X$?

Written assignment 5:

Let $Y = \{3\}$.

   a.   What is 2^Y?

   b.   What is $2^{\left(2^Y\right)}$?

Remember that you are dealing with power sets, so the answers must be sets and NOT numbers!