Math 330 Section 5 - Fall 2023 - Homework 03

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes:
ch.2.1 – 2.4, ch.3.1 – 3.4 until Def.3.13 (Absolute value).

B/G (Beck/Geoghegan) Textbook:
ch.1 – 2.2

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 4:

a. Read carefully the remainder of MF ch.3.

Reading assignment 2 - due: Wednesday, September 6:

a. Read very carefully B/G ch.3 on logic. Review it if Prof. Biddle discussed it when he was substituting for me in the previous week. It is extremely short and covers about all I'll teach you on the subject with the exception of truth tables.
b. Skim MF ch.4.1 - 4.4, just so you have an idea what's in there. Note that I have marked all of ch.4 as optional, but you will be tested on B/G ch.3!

Reading assignment 3 - due Friday, September 8:

a. Skim the remainder of MF ch.4, but look a little bit more closely at ch.4.5.4 (Quantifiers and Negation).
b. Carefully read MF ch.5 through ch.5.2.3. You already encountered much of the material on functions in ch.2.3.

Written assignments are on the next page.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:
Let $(R, \oplus, \odot)$ be an integral domain. Use anything up-to and including MF prop. 3.27 to prove MF prop.3.28: Let $x \in R$. If $x \odot x = x$ then $x = 0$ or $x = 1$.

Hint: Prove the following: If $x \odot x = x$ **and** $x \neq 0$ then $x = 1$. **Why is that enough**?

Written assignment 2: TOUGH! Get started EARLY!
Let $(R, \oplus, \odot, P)$ be an ordered integral domain. Use anything up-to and including MF prop. 3.34 to prove MF prop.3.35: The multiplicative unit 1 of R belongs to P.

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

You are **strongly advised** to study the proof of Proposition 3.33 (newly added to MF version 2021-09-01) very thoroughly before working on this problem.