Math 330 Section 5 - Fall 2023 - Homework 06

Published: Monday, September 14, 2023   *Running total:* 28 *points*
Last submission: Friday, September 29, 2023

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes:
     ch.2.1 – 2.4, ch.3, skim ch.4 (optional), ch.5

B/G (Beck/Geoghegan) Textbook:
     ch.1 – 2.2, ch.3

B/K lecture notes:
     ch.1.1 (Introduction to sets) (optional)
     ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 18:

- **a.** Read MF ch.2.7 (induction)
- **b.** Read very carefully MF ch.6.1. Proofs by induction will be on each major exam!
- **c.** Carefully read B/G ch.2.3., in particular, the fully worked proofs by induction. That's optional, but highly recommended.

Reading assignment 2 - due: Wednesday, September 20:

- **a.** Carefully read MF ch.6.2 – 6.3. Note that this is connected to group homomorphisms (end of ch.3.1)

Reading assignment 3 - due Friday, September 22:

- **a.** Carefully read MF ch.6.4.
- **c.** The strong students are encouraged to read ch.6.5 for its not quite easy proofs by induction.

Written assignments are on the next page.

Written assignment 1:
Let $X, Y \neq \emptyset$ and $f : X \to Y$.

(a) Prove that $R := \{(x, x') \in X \times X : f(x) = f(x')\}$ is an equivalence relation on X.

(b) For the special case $f : \mathbb{R} \to \mathbb{R}; \quad x \to x^2$ compute the equivalence classes $[2], [0], [-2]$ for this equivalence relation.

One point each for **(a)** and **(b)**!!

Written assignment 2:
Prove formulas (5.15) and (5.16) of Proposition 5.3: Let $f : X \to Y$. Then

(a) (5.15) $A_1 \subseteq A_2 \subseteq X \Rightarrow f(A_1) \subseteq f(A_2)$
(b) (5.16) $B_1 \subseteq B_2 \subseteq Y \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

One point each for **(a)** and **(b)**!!