Math 330 Section 5 - Fall 2023 - Homework 08

Published: Saturday, September 30, 2023 Last submission: Friday, October 13, 2023 Running total: 31 points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes: ch.2.1 – 2.7, ch.3, skim ch.4 (optional), ch.5 - 6.12

B/G (Beck/Geoghegan) Textbook (optional, EXCEPT for ch.3 on logic): ch.1 – 3, ch.5 – 6

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional) ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, October 2:

- **a.** Carefully read the remainder of MF ch.6 (very little)
- **b.** Review B/G ch.7.1 (covered in MF ch.6.13) and skim B/G ch.7.2 until before Thm.7.17 (covered in MF ch.6.13). Skip the remainder

Reading assignment 2 - due: Wednesday, October 4:

• Study for the midterm!

Reading assignment 3 - due Friday, October 6:

a. Read very carefully MF ch.7.1 and 7.2. There are a lot of definitions to be learned by rote (and understood by thinking).

Written assignment:

Prove MF Prop. 6.7(a) by induction on p: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in an ordered integral domain $R = (R, \oplus, \odot, P)$, and let $m, n, p \in \mathbb{Z}$ be indices such that $m \leq n < p$. Then

$$\sum_{j=m}^p x_j = \sum_{j=m}^n x_j \oplus \sum_{j=n+1}^p x_j.$$

Hint: Recall the proof of case 2 of Proposition 6.2 ($e(m + n = E(m) \oplus e(n)$) where two variables m, n are involved but m was assumed to be FIXED (but arbitrary) and induction was done only on n. Here you deal with three variables, but only one of them needs to be used for induction. Your life will be easiest if you do induction on p. Think carefully about the base case: What p is minimal for the given $m \le n$?