Math 330 Section 5 - Fall 2023 - Homework 13

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Update November 14, 2023

Replaced reading of Review B/G ch.10.3. – ch.12 with MF ch.11.2.1 (remainder) – 11.2.2

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes:
ch.2.1 – 2.7, ch.3, skim ch.4 (optional), ch.5 - ch.10.2, skip ch.10.3, ch.11.1 - ch.11.2 through ch.11.2.1, Example 11.11

B/G (Beck/Geoghegan) Textbook (optional, EXCEPT for ch.3 on logic):
ch.1 – 3, ch.5 – 7

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, November 13:

- Review B/G ch.8 - ch.10.3.

UPDATED Reading assignment 2 - due Wednesday, November 15:

a. Moved to HW 14: Review B/G ch.10.3. – ch.12, but skip ch.11.3.
b. INSTEAD: Carefully read the remainder of MF ch.11.1 and MF ch. 11.2.2.

UPDATED Reading assignment 3 - due Friday, November 17:

a. The strong students are invited to peruse the optional chapter MF 11.2.3.
b. Read carefully MF ch.12.1 and ch.12.2. It is crucial that you become familiar with the examples given there You will have massive problems with metric and topological spaces if you did not study MF ch.9.3.
c. Read carefully MF ch.12.3. What does an open set in $\mathbb{R}$ with $d(x,y) = |y-x|$ look like?

Be sure to read pages 2 and 3!

Supplementary instructions for reading MF ch.12:

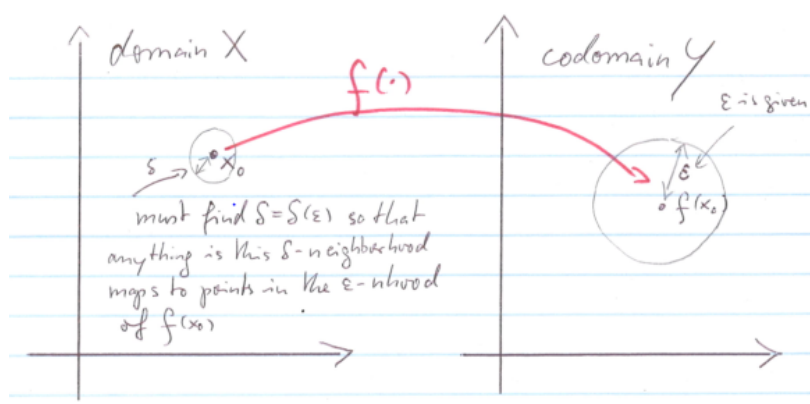
When you read or reread any topics in those chapters then the following is good advice:

a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

- open sets and neighborhoods (ch.12.1.3)
- convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
- metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What is $N_\varepsilon^A(x_j)$?
- Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.12.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 0.1: ε-δ continuity



Written assignments on page 3

Written assignment 1: Prove formula (9.14) of prop.9.11: Let X be a nonempty set and $\varphi, \psi : X \to \mathbb{R}$. Let $\emptyset \neq A \subseteq X$. Then

$$\inf\{\varphi(x) + \psi(x) : x \in A\} \ \geqq\ \inf\{\varphi(y) : y \in A\} \ +\ \inf\{\psi(z) : z \in A\}.$$

Do the proof by modifying the proof of formula (9.13). Follow that proof as closely as possible!
You are **NOT ALLOWED** to apply formula (9.13) to $-\varphi$ and $-\psi$.

Written assignment 2: Prove MF prop.9.18**(b)**: If y_n is a sequence of real numbers that is non-increasing, i.e., $y_n \geqq y_{n+1}$ for all n, and bounded below, then $\lim\limits_{n\to\infty} y_n$ exists and coincides with $\inf\{y_n : n \in \mathbb{N}\}$.

Do the proof by modifying the proof of prop.9.18**(a)**. You are **NOT ALLOWED** to apply prop.9.18**(a)** to the sequence $x_n := -y_n$!