

Math 330 Section 5 - Fall 2023 - Homework 14

Published: Thursday, November 14, 2023
Last submission: Monday, December 1, 2023

Running total: 44 points

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete before the first one of this HW.

MF lecture notes:

ch.2.1 – 2.7, ch.3, skim ch.4 (optional), ch.5 - ch.10.2, skip ch.10.3, ch.11.1 - ch.12.3; skip ch.11.2.3 (optional)

B/G (Beck/Geoghegan) Textbook (optional, EXCEPT for ch.3 on logic):

ch.1 – 3, ch.5 – 10.2

B/K lecture notes:

ch.1.1 (Introduction to sets) (optional)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, November 20:

- Review B/G ch.10.3. – ch.12, but skip ch.11.3. (was originally scheduled in HW 13)

Reading assignment 2 - due Wednesday, November 22 (Thanksgiving break):

- a. Carefully read MF ch.12.4 and 12.5.
- b. The better students are encouraged to at least skim ch.12.6

Reading assignment 3 - due Friday, November 24 (Thanksgiving break):

- a. Review one more time MF ch.11.2.2 and ch.12.1 – 12.3.

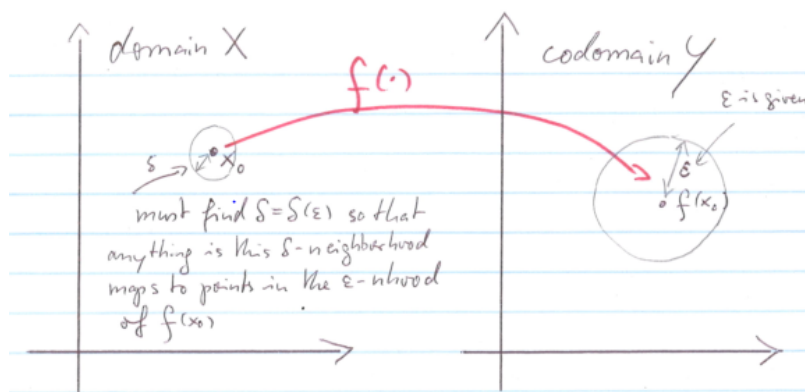
Be sure to read pages 2 and 3!

Supplementary instructions for reading MF ch.12:

When you read or reread any topics in those chapters then the following is good advice:

- a. MF ch.12.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d|_{\|\cdot\|_2})$ and $(\mathcal{B}(X, \mathbb{R}), d|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for
 - open sets and neighborhoods (ch.12.1.3)
 - convergence, expressed with nhoods (the end of def.12.10 in ch.12.1.4)
 - metric and topological subspaces (ch.12.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \uplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε -nhoods (circles with radius ε about x_j) so that some circles are entirely in A , one with $x_j \in A_1$ which reaches into A° but not into A_2 , and one with $x_j \in A_2$ which reaches both into A° and A_1 . What is $N_\varepsilon^A(x_j)$?
 - Contact points, closed sets and closures (ch.12.1.8): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is \bar{B} ?
 Draw points "completely inside" B , others "completely outside" B , and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B , i.e., which ones are interior points of B ? Which ones can you surround by circles that entirely stay outside the closure of B , i.e., which ones are entirely within \bar{B}° ? Use those pictures to visualize the definitions in this chapter and thm 12.6 and thm.12.7.
 - Now repeat that exercise with an additional set A which is meant to be a metric subspace of \mathbb{R}^2 .
- b. MF ch.12.2 (Continuity): Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 0.1: ε - δ continuity



Written assignments on page 3

Written assignment 1:

Let $f(x) = x^2$. Prove by use of “ ε - δ continuity” that f is continuous at $x_0 = 1$. You MUST work with ε - δ continuity (thm.9.7) **NOT WITH SEQUENCE CONTINUITY**, and you cannot use any “advanced” knowledge such as the product of continuous functions being continuous, etc.

This assignment is worth up to 3 points. Partial credit will be given and you can turn it in repeatedly.

Special instructions for assignment 1: Turn in your scratchpaper where you solve for δ (see the hints below).

Hints:

- a. What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0)) < \varepsilon$ translate to?
- b. $x^2 - 1 = (x + 1)(x - 1)$.
- c. Do the following on scratch paper: Work your way backward by establishing a relationship between $\varepsilon > 0$ and δ and then “solving for δ ” That part should not be in your official proof.
- c1. Only small neighborhoods matter (see Proposition 9.24 at the end of the chapter on convergence and continuity.): Given $0 < \varepsilon < 1$ try to find δ that works for such ε . Restrict your search to $\delta < 1$. What kind of bounds do you get for $|x^2 - 1|$, $|x + 1|$, $|x - 1|$ if $0 < \delta < 1$? In particular what kind of bounds do you get for $|x + 1|$?
- c2. Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider ε as given? You’ll get the answer by “solving $|f(x) - f(x_0)| \leq 3\delta$ for δ ”.
- c3. All of the above was done under the assumption that $\delta < 1$. Satisfy it by replacing δ with $\delta' := \min(\delta, 1)$
- d. Only now you are ready to construct an acceptable proof: Let $\varepsilon > 0$, $\delta := \dots$, and $\delta' := \min(\delta, 1)$. Then

Written assignment 2: Prove MF Thm. 9.8: If $m \in [0, \infty[_\mathbb{Z}$ is not a perfect square then \sqrt{m} is irrational.

Hint: Work with lowest term representations. See the proof that $\sqrt{2}$ is irrational.

No partial credit for this one!