Math 330 Section 1 - Fall 2024 - Homework 05

Published: Thursday, September 12, 2024 Running total: 24 points
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Status - Reading Assignments:

The reading assignments you were asked to complete before the first one of this HW are:

MF lecture notes:
ch.1; ch.2.1 - 2.6, ch.3; skim ch.4; ch.5, ch.6.1 - 6.7

B/G (Beck/Geoghegan) Textbook:
ch.2.1 – 2.3, ch.5

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments: None!

Written assignments are on the next page.

Written assignments:

These written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

Written assignment 1:

Injectivity and Surjectivity

- Let $f : \mathbb{R} \longrightarrow [0, \infty[; \quad x \mapsto x^2$.
- Let $g : [0, \infty[\longrightarrow [0, \infty[; \quad x \mapsto x^2$.
  In other words, g is same function as f as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with **true** or **false**.

a.	f is surjective	**c.**	g is surjective
b.	f is injective	**d.**	g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2: Let $X := \{-2, 2\}, Y := \{4\}$, and $f : X \longrightarrow Y$ defined as $f(x) = x^2$. Find $A \subseteq X$ such that $f(A^{\complement}) \neq f(A)^{\complement}$.

Compute both $f(A^{\complement})$ and $f(A)^{\complement}$.

Written assignment 3: Here, all intervals are understood to be intervals of **real** numbers (not of integers)! Let $f :]-10, 10[\longrightarrow \mathbb{R}; \quad x \mapsto x^2$.

a. what is the range of f? **b.** Is f injective? **c.** Is f surjective?

d. $f(\{1\} \cup [4, 6]) = ?$ **e.** $f([2, 5]) \cap f([4, 7]) = ?$ **f.** $f^{-1}([4, 25]) \cap f^{-1}([16, 49]) = ?$

Hint: For **d, e, f**, review examples 5.24–5.27.

Written assignment 4:

You have learned in MF ch.5 that

injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective.

The following illustrates that the reverse is not necessarily true.

Let $A := \{a\}$ and $B := \{b_1, b_2\}$. Assume that $b_1 \neq b_2$. Find functions $f : A \to B$ and $g : B \to A$ which satisfy the following:

- The composition $h := g \circ f : A \to A$ is bijective.
- It is **not true** that both f, g are injective.
- It is **not true** that both f, g are surjective.

Hint: There are not a whole lot of possibilities. Draw all possible candidates for f and g in arrow notation as you see in MF example 5.19. There are only very few choices!